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Notes, Comments, and Letters to the Editor

Bidding Off the Wall: Why Reserve Prices May Be Kept Secret

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This note shows by means of an example that in a common-value auction a seller with a random reservation value can increase her ex ante expected profits by following a policy of conducting an auction in which her reserve price is kept secret compared to an auction in which the reserve price is announced. By keeping the reserve price secret, the seller is able to encourage greater participation from the bidders and can, therefore, increase the linkage of the price paid to the value of the purchased object. *Journal of Economic Literature* Classification Numbers: D82, D44.

1. INTRODUCTION

Since the publication of a well-known paper by Milgrom and Weber [6], it has been recognized that if a seller of a good at a common-value auction has private information about an object to be sold, she can increase her expected profits by following a policy of credibly revealing the information. By making relevant information public, the seller is able to alleviate some of the costs due to the winner's curse and thereby increase the average bid. In view of this wisdom, it has seemed to be a puzzle that in many auctions, sellers will typically not announce the reserve price in advance—a phenomenon documented by at least three recent papers (Ashenfelter [1], Hendricks and Porter [3], and Elyakime *et al.* [2]). At auctions of fine wines and art, auctioneers will generate phantom bids "off the wall" or "from the chandelier" in order to keep the object in-house

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when the bidding from the floor is not high enough to warrant selling the good.[1]

Does such behavior violate the principle of the optimality of information revelation? This paper shows that a policy of keeping private reserve prices can be revenue-enhancing for a seller in a common-value auction. The conclusion, of course, does not overturn the standard wisdom. Instead it follows from it. The announcement of a reserve price may have an inhibiting effect on the participation of bidders in a given auction—for some potential bidders, the only possibility of winning is to win at the reserve price and such an event may occur only when the object is not worth purchasing. This possibility will discourage some bidders from participating. As a result, their information does not play a role in the process of the auction even though it may be relevant for the valuation of other bidders. The consequence is to prevent some sales from being made even though the aggregate information would imply that a transaction should occur. This note illustrates that in a Bayesian game in which the reservation price is kept secret, the seller can induce more participation by bidders. In general, bidders submit lower bids since it is not known whether the rival price is that of another bidder or the seller but the seller may be willing to incur this cost if the policy encourages more bidders to participate and therefore induces a greater aggregation of information.

To see the inhibiting effect of a reserve price, consider a simple two-bidder auction with affiliated values. Let $v(x_i, x_j)$ denote the expected value of the object to bidder i with signal x_i conditional on bidder j's signal, x_j. With no reserve price, an equilibrium profile is for each bidder i to bid $b(x_i) = v(x_i, x_i)$. Suppose, though, that a non-trivial reserve price, r, is posted and let $d(r)$ solve $E[v(d(r), x_j) \mid x_j \leqslant d(r)] = r$ such that $d(r)$ lies above the bottom of the support of x_i. Let $c(r)$ satisfy $v(c(r), c(r)) = r$. Strict affiliation then implies that $d(r) > c(r)$. Monotonicity and continuity of the $v(\cdot, \cdot)$ function implies that no bidder with a signal in $(c(r), d(r))$ will submit a serious bid. The consequence is that for any r, with positive probability, some trades will fail to occur even when the aggregate market information should have generated a bidder value above r. The following simple example shows that this inefficiency can reduce the seller's ex ante profits.

[1] For an interesting discussion of this phenomenon, see Ashenfelter [1]. According to Ashenfelter, "If you sit through an auction you will find that every item is hammered down and treated as if it were sold.... In short, the auctioneers do not reveal the reserve price and they make it as difficult as they can for bidders to infer it."

2. Equilibrium with an Announced Reserve Price

For ease of analysis, this paper will concentrate on a second-price auction. While second-price auction and English auctions generally are not equivalent, the effect highlighted in this paper—that is, the inhibiting effect that the announcement of a reserve price will have on participation rates—will only be stronger in an English auction where the participation of bidders conveys even more information. Furthermore, the speed at which wine auctions, for example, proceed suggests that most participants are not actually aware of more than the fact that some other bidders are active. They rarely can tell if and at what price bidders drop out. The practical consequences of focusing on a second-price auction seem relatively harmless.

This paper examines a specific class of common values auction games with one seller with use value s and n bidders. The seller's use value is assumed to take on the value of s_l with probability β and to be distributed uniformly over the interval $[s_l, s_h]$ with probability $1-\beta$. Bidders observe private signals x_i. Each x_i is independently and uniformly distributed over the interval $[0, 1]$. Bidder i's (ex post) valuation is given by $v_i(x_i, x_{-i}) = \alpha x_i + (1-\alpha)(\sum_{j\neq 1}^{n} x_j)/(n-1)$. It is assumed that the reservation value of the seller does not affect the value of the object to the buyers.[2]

While this model is not the conventional affiliated values model in the formulation of Milgrom and Weber [6] it shares with that model the feature that the expected value of $v_i(x_i, x_{-i})$ rises with more favorable information about any of the informational variables. This version provides some computational conveniences not generally available in the standard framework. The variable α serves as a means of parametrizing the degree of common values present—$\alpha = 1$ represents a pure private values model, $\alpha = 1/n$ a pure common values model. The linearity of valuations and the uniform distribution will be responsible for generating very simple linear bidding functions.[3]

In an announced reserve price auction, the seller announces a reserve price, r_A, and commits herself to sell the object only at a price r_A or higher. If the second highest bid exceeds r_A then the highest bidder buys the object

[2] This is not a very strong restriction since it may be presumed that, as in wine auctions, information that the seller has which is relevant to the buyers' valuations and can be made public has been made public already. Milgrom and Weber [6] show that such a policy is revenue-enhancing.

[3] One environment generating such a formulation is a situation in which the object to be sold is used as an input in a downstream market with positive externalities. For example, suppose the final product sells at a price of 1, and the object sold can be transformed into the final output at a ratio of u_i. Each signal, x_i, essentially represents a form of linear cost-saving technology which cannot be kept fully private. The α parameter is a measure of the degree of spillover of the technology available after the auction but before production.

at a price equal to the second highest bid but if the second highest bid is less than r_A and the highest bid exceeds r_A the high bidder purchases the object at the price r_A. For any announced reserve price, r_A, the equilibrium bidding strategies of the buyers can be explicitly computed. Let $d(r_A)$ be the lowest type of bidder who submits a bid greater than or equal to r_A. For notational ease, we focus on bidder 1. The term Y_1 denotes the random variable which is the highest of the $n-1$ other signals. If the price paid is r_A, then all that is known is that all other bidders have bid less than r_A. Thus, the expected value of the object to an agent who observes signal $x_1 = d(r_A)$ and wins at the reserve price is $E[v \mid x_1 = d(r_A), Y_1 \leqslant d(r_A)] = \alpha d(r_A) + (1-\alpha)\, d(r_A)/2$. Setting this equal to r_A implies that

$$d(r_A) = r_A(2/(1+\alpha)) \equiv r_A d.$$

Winning at the reserve price and winning above the reserve price have different informational consequences. When the price is strictly above r_A, then the second highest bid is known precisely. Suppose that all other bidders follow a bidding strategy determined by a strictly monotonic bidding function, $b(\cdot)$. The expected value of the object to an agent when a signal $x_1 = x$ is observed and the object is won at a price, p, above the reserve price is

$$E[v_1(x, x_{-1}) \mid x_1 = x,\ Y_1 = b^{-1}(p)] = \alpha x + n\frac{1-\alpha}{n-1}\frac{b^{-1}(p)}{2}.$$

Standard arguments (for example, see Milgrom and Weber [6]) can be used to show that for any reserve price r_A, a Bayesian Nash equilibrium profile of bids is given by

$$b(x) = \begin{cases} x\left(\dfrac{1+\alpha}{2} + \dfrac{1-\alpha}{2(n-1)}\right), & \text{if } \quad x \geqslant r_A\dfrac{2}{(1+\alpha)} \\ < r_A & \text{otherwise.} \end{cases}$$

Agents with relatively high signals submit bids corresponding to the standard Nash equilibria in second-price auctions. Agents with low signals do not submit serious bids. The cut-off is found by determining the signal level $d(r_A)$ such that if an agent observes $x = d(r_A)$ and wins at price r_A, the expected value to him of the object conditional on the information that all other bidders' signals lie below $d(r_A)$ is exactly r_A. Observe that for $\alpha < 1$, $2/(1+\alpha) > 1$; thus some bidders with signal y fail to participate even though $E[v_1(x_1, x_{-1}) \mid x_1 = y, Y_1 = y] > r_A$.

In general, the seller will wish to set her reserve price strictly higher than her use value, s, in order to extract a higher surplus from the bidders. Let X_i be the ith order statistic of all n bidders. In a second-price auction, for a seller of type s the expected return from an announced reserve price of r_A is given by

$$\begin{aligned}\Pi_A(r_A \mid s) = r_A \operatorname{Prob}[X_1 \geqslant d(r_A) \text{ and } X_2 \leqslant d(r_A)] \\ + E[b(X_2) \mid X_2 \geqslant d(r_A)] \operatorname{Prob}[X_2 \geqslant d(r_A)] \\ - s \operatorname{Prob}[X_1 \geqslant d(r_A)].\end{aligned}$$

She obtains r_A when only X_1 exceeds the cut-off point $d(r_A)$ and obtains the second highest type's bid when they both exceed dr_A. Analytic solution for the optimal choice of r_A are in general rare in common value auctions. The advantage of the particular choice of the payoffs and information structure is that it allows an explicit computation of the optimal reserve price.

LEMMA 1. *For any s, the choice of r_A which maximizes the seller's expected revenue must satisfy*

$$dr_A(s) = \begin{cases} \min[1, ((s+\alpha)2)/(1+3\alpha)], & \text{if } s \geqslant -\alpha \\ 0 & \text{otherwise.} \end{cases}$$

Proof of Lemma. Maximize $\Pi_A(r_A \mid s)$ with respect to r_A. Q.E.D.

Observe that when s exceeds $1/d$, no bidder type would submit a bid which the seller would ever accept. Substituting $r_A(s)$ into $\Pi_A(r_A \mid s)$ yields the expected utility $\Pi_A(s)$ for a seller of type s from an auction in which the optimally chosen reserve price is announced.

3. A BETTER WAY?

If the seller's use value s were common knowledge, then whether or not a reserve price was announced would make no difference in an auction—bidders would simply compute the seller's optimal r and behave as if it were announced. If s is random and only the seller observes s, she could, if she desired, announce the reserve price $r_A(s)$ and conduct the auction as in Section 1. Alternatively, she could tell the reserve price to the auctioneer and instruct him not to reveal it to the bidders. If the bidding does not go high enough, the auctioneer will have to create bids off the wall. Again, sales occur whenever the highest bid exceeds the reserve price but now that the reserve price is not known, it will not be common knowledge whether the winning price is that of a bidder or that of the seller.

A secret reserve price auction is a Bayesian game between the seller and the bidders. In a Bayesian Nash equilibrium of this game, given the behaviour of the bidders, each seller type must choose her best reserve price, r_S and given this choice rule of seller types (and the prior distribution of seller types) bidders must choose their optimal bidding strategy. In general, the characterization of an equilibrium of this game is not tractable; however, for the specification described in Section 1 and for the particular seller distribution, we can arrive at a closed form solution of the game.

Lemma 2 characterizes the best response of a seller of type s when n bidders follow any linear bidding strategy $b(x) = mx$.

LEMMA 2. *If n bidders each with private information x drawn independently from the uniform $[0, 1]$ distribution submit bids mx in the secret reserve price auction, then the optimal reserve price for a seller of type $s \geqslant -m$ is $r_S(s) = (m + s)/2$.*

Proof of Lemma. A seller of type s who sets a reserve price of r receives an expected profit of

$$\Pi_S(r_S \mid s) = r_S n\left(1 - \frac{r_S}{m}\right)\left(\frac{r_S}{m}\right)^{n-1}$$
$$+ m \int_{r_S/m}^{1} yn(n-1)(1-y)\, y^{n-2}\, dy - s\left(1 - \left(\frac{r_S}{m}\right)^{n}\right).$$

Maximizing this expression with respect to r_S yields the desired result.
Q.E.D.

For the remainder of this section, I assume that $s_l = -m$ and $s_h = m$. This assumption allows for tractable solutions of the secret reserve price game. If the seller types choose the optimal reserve price according to Lemma 2, but do not announce s or r, then from the point of view of the bidders, the reserve price is random and either is 0 with probability β or, with probability $1 - \beta$, is uniformly distributed over $[0, m]$.

This choice of distributions yields a particularly simple updating rule. Consider the viewpoint of bidder 1. Suppose that the seller types follow the strategy in Lemma 2 and that the other bidder follows a linear bidding strategy, $b(x) = mx$. This implies that with probability one, the winning price will exceed 0. Suppose that bidder 1 wins at a price, p. There are two possible events. In the first which occurs with probability β, the seller obtains a draw, $s = -m$. In this case, the probability p was submitted by the seller is 0. In the second event which occurs with probability $1 - \beta$, the seller receives a draw of $s > -m$ and submits a bid which, conditional on this event, is uniform over $[0, m]$. In this case, the probability that p was submitted by the seller is $1/n$, or equal to the probability it was submitted by another bidder. Therefore, if ϕ is the probability that a winning price was the seller's reserve price, $\phi = (1-\beta)/n$. Note that ϕ is also the probability that the highest of the other bidder's valuation lies below p/m. The main contribution of the particular assumptions about the distributions lies in the implication that ϕ is independent of the winning price, p. Define

$$m \equiv \frac{1+\alpha}{2} + \frac{(1-\alpha)(1-\phi)}{(n-1)\,2}.$$

THEOREM 3. *The profile of strategies, $b(x) = mx$ for all bidders and $r_S(s) = (m+s)/2$ for the seller, is a Bayesian Nash equilibrium of the second-price auction game when the seller's reserve price is private.*

Proof. Let p be the (random) price paid. Assume that all other $n-1$ bidders follow the strategy $b(x) = mx$ and consider bidder 1 with signal x. The expected payoff from winning the object at a bid, p, is

$$u(x, p) = \alpha x + \frac{1-\alpha}{n-1}\left[\frac{\phi}{2} + (1-\phi) + \frac{n-2}{2}\right]\frac{p}{m}.$$

In a second-price auction, a submitted bid affects the probability of purchasing the object and the support of the distribution of prices, p, that are paid. Since $u(\cdot, \cdot)$ is increasing in both its arguments and by definition of m, $u(x, mx) = mx$, if the price paid is above mx, say $p = my = u(y, my) > u(x, my) > u(x, mx)$, the buyer's net expected utility conditonal on winning at a price above mx is strictly negative. Similarly, if the price paid is strictly less than mx, the bidder's expected utility conditional on winning at a price between p and mx is strictly positive. Therefore, given the presumed behavior of the seller and the other bidders, $b(x) = mx$ is a best response for any bidder of type x. Given that bidders choose mx, Lemma 2 shows that a reserve price strategy $r(s) = (s+m)/2$ is a best response on the part of the seller which completes the proof. Q.E.D.

The expected return of the secret reserve price auction to a seller with use value, s, can be found by using the definition of m for the buyers and substituting the optimal secret reserve price of the seller from Lemma 2 into $\Pi_S(r_A(s) \mid s)$.

Observe that *all* bidder types submit bids in this auction but that bidders typically shade their bid down in order to account for the possibility that the object is won at a reserve price bid instead of a buyer's offer. If r was known to be 0, that is, in the standard second-price auction form, a Nash equilibrium profile of bids is

$$b(x) = \frac{1+\alpha}{2} + \frac{(1-\alpha)}{(n-1)\,2} > mx.$$

On the other hand, if r is greater than 0 and is announced, then Theorem 3 shows that a significant and ex post inefficient measure of bidders do not participate in the auction. These are the two effects that a seller wishes to trade off in the choice of an auction policy.

Which policy, an announced or a secret reserve price, yields the seller the highest expected revenue on average can be found by substituting $r_A(s)$ and $r_S(s)$ into the profit function, yielding $\Pi_A(r_A(s)|s)$ and $\Pi_S(r_S(s)|s)$ as expected profits for any given n, α, β and integrating against s to yield Π_A, the expected gains from an announced price auction and Π_S, the expected gains from a secret price auction.

Table I shows the values of these profits for various values of α and n with $\beta = 0$ and $\beta = 0.5$. If α is less than 1, some affiliation among valuations is present and the secret reserve price may be more profitable than an announced price. With a higher number of bidders, the advantages of keeping the reserve price secret remain but are diminished. Presumably, this advantage disappears as the number of bidders becomes large since for any reserve price, the probability that at least two bidders have values above the reserve price approaches 1. Note that when $\beta = 0$, an announced reserve price auction is always preferable, suggesting that it is in environments where there is a relatively high weight on lower type sellers that the policy of keeping the reserve price secret is optimal.

For $\alpha = 1$, the two policies generate the same revenue since the common-value element is absent. This feature, that in the pure private-value environment keeping the reserve price secret yields no advantage, is a general one for second-price auctions. This is because bidder's equilibrium strategies are easily shown to depend on their own signal alone. It is often useful to be able to determine if a given auction is characterized by affiliated or private values. The fact that reserve prices are kept secret may be seen as evidence supporting the presence of a common-value element in an auction.

TABLE I

$\Pi_A - \Pi_S$

	$\beta = 0$			$\beta = 0.5$		
	$n = 2$	$n = 3$	$n = 4$	$n = 2$	$n = 3$	$n = 4$
$\alpha = 0.5$	0.008	0.004	0.002	−0.004	−0.001	0
$\alpha = 0.6$	0.005	0.002	0.001	−0.004	−0.001	−0.001
$\alpha = 0.7$	0.003	0.001	0.001	−0.004	−0.001	−0.001
$\alpha = 0.8$	0.001	0.001	0	−0.003	−0.001	−0.001
$\alpha = 0.9$	0	0	0	−0.002	−0.001	0
$\alpha = 1$	0	0	0	0	0	0

4. Discussion

The welfare analysis for the seller is done in an ex ante context. It is important, therefore, that the seller be able to commit to a policy of always revealing or never revealing before she learns her private information. This is because whenever the seller's type is in fact very low, she has an incentive to announce the fact, ex post, since the resulting low reserve price will not discourage many bidders in any case. The role of the auctioneer may be seen, in part, as serving the function of such a commitment device. Of course, the question then arises as to whether a sufficient incentive emerges for deviant auction houses to operate with announced reserve prices and thus attract the low-type sellers.

Other forms of common valuations can be put into this framework and yield similar results. What is important is the exploitation of what Milgrom [5] calls the "linkage principle." The announcement of a reserve price which may be above the bids of some buyers breaks part of the linkage between the price paid and the value of the object. Keeping the reserve price secret is a way of restoring this linkage by inducting greater participation and thereby increasing the seller's profits. Mathematically, this result is reminiscent of a result of McAfee and McMillan [4] which shows that maintaining uncertainty about the number of bidders can increase seller revenue. In both cases the induced uncertainty can be used to exploit a convexity in the seller revenue function to increase expected revenues.

The robustness of this example is difficult to assess completely since the choice of the distribution of the seller's type (which is important to order to yield a tractable solution to the Bayesian game) also plays a role in the determination of ex ante profits. The particular distribution used here has the property of putting high weight on low seller types. Nevertheless, the example highlights an important factor to consider in naming reserve prices. The announcement of a reserve price may inhibit the ability of the auction mechanism to aggregate the information of other bidders and, as a result, lower expected revenues.

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