# Self-Calibrating Priors Do Not Exist

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It is shown that self-calibrating priors in the sense of Dawid (1982a) do not exist.

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Dawid (1982a) gave a stimulating discussion of the behavior of a coherent Bayesian forecaster faced with the task of sequentially predicting the probabilities of events $S_i$—for example, rain on day $i$ ($i = 1, 2, \ldots$). The forecast $\hat{Y}_i$ of the indicator variable $Y_i$ of $S_i$ is made on day $i - 1$, and the forecaster has available at that time the history of previous forecasts and their outcomes, but not the outcome on day $i$. Thus, $\hat{Y}_i$, but not $Y_i$, is $B_{i-1}$-measurable, where the $\sigma$ field $B_{i-1}$, denoting the forecaster's state of knowledge on day $i - 1$, includes that generated by $\{Y_1, Y_2, \ldots, Y_{i-1}\}$.

Dawid shows that the forecaster expects—that is, with probability one according to his (initial) prior distribution $\Pi$ over $B_\infty = V_{i=0}^{\infty} B_i$—the following *calibration property* to hold: Among days on which the forecast probability of rain is $a$, the limiting relative frequency of actual rainy days will also equal $a$. In fact, using martingale arguments, Dawid proves a much stronger result, which need not concern us here.

Dawid notes that a forecaster cannot adjust his future predictions to correct for past empirical miscalibration without sacrificing coherence. He stated, "The conflict between calibration and coherence could be avoided only by a distribution $\Pi$ that was not even potentially miscalibrated" and "it seems doubtful although not impossible that such a coherent, self-calibrating distribution could exist" (Dawid 1982a; p. 609).

The object of this article is to point out that no prior distribution $\Pi$ can be self-calibrating in this sense. Dawid's definition would require the calibration property to hold for all sequences $\{Y_1, Y_2, \ldots\}$ and hence for all probability models $P$ generating such a sequence. To show that $\Pi$ is not self-calibrating, it suffices to exhibit one such $P$ for which the property does not hold. Such a $P$ (depending on $\Pi$) can easily be produced. For example, set

$$P(S_i \mid B_{i-1}) = f\{\Pi(S_i \mid B_{i-1})\},$$

where the function $f[0, 1] \rightarrow [0, 1]$ is defined by $f(x) = x + \frac{1}{2}$ $(0 \le x \le \frac{1}{2})$, $f(x) = 1 - x$ $(\frac{1}{2} < x \le 1)$. Consider the random subsequence of days $(I_1, I_2, \ldots)$ for which $\hat{Y}_i = a$. Then under $P$ the variables $Y_{I_1}, Y_{I_2}, \ldots,$ form a Bernoulli sequence with $P(Y_{I_k} = 1) = f(a)$. If nature adopts the strategy $P$, the actual relative frequency of rainy days among days with forecast probability $a$ will approach $f(a) \neq a$.

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## REFERENCE

Dawid, A. P. (1982a), "The Well-Calibrated Bayesian" (with comments), *Journal of the American Statistical Association*, 77, 605–613.

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