Rank-Order Tournaments as Optimum Labor Contracts

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This paper analyzes compensation schemes which pay according to an individual's ordinal rank in an organization rather than his output level. When workers are risk neutral, it is shown that wages based upon rank induce the same efficient allocation of resources as an incentive reward scheme based on individual output levels. Under some circumstances, risk-averse workers actually prefer to be paid on the basis of rank. In addition, if workers are heterogeneous in ability, low-quality workers attempt to contaminate high-quality firms, resulting in adverse selection. However, if ability is known in advance, a competitive handicapping structure exists which allows all workers to compete efficiently in the same organization.

I. Introduction

It is a familiar proposition that under competitive conditions workers are paid the value of their marginal products. In this paper we show that competitive lotteries are often efficient and sometimes superior to more familiar compensation schemes. For example, the large salaries of executives may provide incentives for all individuals in the firm who, with hard labor, may win one of the coveted top positions.

This paper addresses the relation between compensation and incentives in the presence of costly monitoring of workers' efforts and

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output. A wide variety of incentive payment schemes are used in practice. Simple piece rates, which have been extensively analyzed (see, e.g., Cheung 1969; Stiglitz 1975; Mirrlees 1976), gear payment to output. We consider a rank-order payment scheme which has not been analyzed but which seems to be prevalent in many labor contracts. This scheme pays prizes to the winners and losers of labor market contests. The main difference between prizes and other incentive schemes is that in a contest earnings depend on the rank order of contestants and not on "distance." That is, salaries are not contingent upon the output level of a particular game, because prizes are fixed in advance. Performance incentives are set by attempts to win the contest. We argue that in many circumstances it is optimal to set up executive compensation along these lines and that certain puzzling features of that market are easily explained in these terms.

Central to this discussion are the conditions under which mechanisms exist for monitoring productivity (Alchian and Demsetz 1972). If inexpensive and reliable monitors of effort are available, then the best compensation scheme is a periodic wage based on input. However, when monitoring is difficult, so that workers can alter their input with less than perfect detection, input-wage schemes invite shirking. The situation often can be improved if compensation is related to a more easily measured output level. In general, input-based pay is preferable because it changes the risk borne by workers in a favorable way. But when monitoring costs are so high that moral hazard is a serious problem, the gain in efficiency from using output-based pay may outweigh the risk-sharing losses. Paying workers on the basis of rank order alters costs of measurement as well as the nature of the risk borne by workers. It is for these reasons that it is sometimes a superior way to bring about an efficient incentive structure.

In the development below we start with the simplest case of risk neutrality to illustrate the basic issues. Then the more general case of risk aversion is treated in Section III. Section IV considers issues of sorting and self-selection when workers are heterogeneous.

II. Piece Rates and Tournaments with Risk Neutrality

To keep things simple and to avoid sequential and dynamic aspects of the problem, we confine attention to a single period in all that follows. Therefore, the reader should think of the incentive problem in terms of career development and lifetime productivity of workers. The worker's (lifetime) output is a random variable whose distribution is controlled by the worker himself. In particular, the worker is allowed to control the mean of the distribution by investing in costly skills

prior to entering the market. However, a given productivity realization also depends on a random factor which is beyond anyone's control. Employers may observe output but cannot ascertain the extent to which it is due to investment expenditure or to good fortune or to both, though workers know their input as well as output. Worker j produces lifetime output q_j according to

$$q_j = \mu_j + \epsilon_j, \tag{1}$$

where μ_j is the level of investment, a measure of skill or average output, chosen by the worker when young and prior to a realization of the random or luck component, ϵ_j. Average skill, μ_j, is produced at cost $C(\mu)$, with $C', C'' > 0$. The random variable ϵ_j is drawn out of a known distribution with zero mean and variance σ^2.[1] Here ϵ is lifetime luck such as life-persistent person-effects or an ability factor, which is revealed very slowly over the worker's lifetime. The crucial assumption is that productivity risk is nondiversifiable by the worker himself. That is another reason for choosing a long period for the analysis. For example, if the period were very short and the random factor was independently distributed across periods, the worker could diversify per period risk by repetition and a savings account to balance off good and bad years. Evidently a persistent person or ability effect cannot be so diversified when it is undiscoverable quickly, as appears true of managerial talent, for example. It is assumed, however, that ϵ is i.i.d. across individuals, so that owners of firms can diversify risk either by pooling workers together in one firm or by holding a portfolio.

To concentrate on incentive aspects of various contractual arrangements, we adopt the simplest technology for firms. Production requires only labor and is additively separable across workers. By virtue of the independence assumptions, managers act as expected value maximizers or as if they were risk neutral. Free entry and a competitive output market set the value of the product at V per unit. Again, these assumptions are adopted to illustrate basic issues in the simplest way. The analysis also applies when there are complementarities among workers in production, which is more realistic but more difficult to exposit.

Piece Rates

The piece rate is very simple to analyze when workers are risk neutral. It involves paying the worker the value of his product. Let r be the

[1] In this paper the worker has no choice over σ. This does not affect the risk-neutral solution but does have an effect if workers are risk averse, since they tend to favor overly cautious strategies. Also, virtually all the results of this paper hold true if the error structure is multiplicative rather than additive.

piece rate. Ignoring discounting, the worker's net income is $rq - C(\mu)$. Risk-neutral workers choose μ to maximize expected net return

$$E[rq - C(\mu)] = r\mu - C(\mu).$$

The necessary condition is $r = C'(\mu)$ or the familiar requirement that investment equates marginal cost and return. On the other hand, the expected profit of a firm is

$$E(Vq - rq) = (V - r)\mu,$$

so free entry and competition for workers imply $r = V$. Consequently

$$V = C'(\mu).$$

The marginal cost of investment equals its social return, yielding the standard result that piece rates are efficient.

Rank-Order Tournaments

We shall consider two-player tournaments in which the rules of the game specify a fixed prize W_1 to the winner and a fixed prize W_2 to the loser. All essential aspects of the problem readily generalize to any number of contestants. A worker's production follows (1), and the winner of the contest is determined by the largest drawing of q. The contest is rank order because the margin of winning does not affect earnings. Contestants precommit their investments early in life, knowing the prizes and the rules of the game, but do not communicate with each other or collude. Notice that even though there are two players in a given match the market is competitive and not oligopolistic, because investment is precommitted and a given player does not know who his opponent will be at the time all decisions are made. Each person plays against the "field."

We seek to determine the competitive prize structure (W_1,W_2). The method proceeds in two steps. First, the prizes W_1 and W_2 are fixed arbitrarily and workers' investment strategies are analyzed. Given these strategies, we then find the pair (W_1,W_2) that maximizes a worker's expected utility, subject to a zero-profit constraint by firms. It will be seen that a worker's incentives to invest increase with the spread between winning and losing prizes, $W_1 - W_2$. Each wants to improve the probability of winning because the return to winning varies with the spread. The firm would always like to increase the spread, *ceteris paribus*, to induce greater investment and higher productivity, because its output and revenue are increased. But as contestants invest more, their costs also rise. That is what limits the spread in equilibrium: Firms offering too large a spread induce excessive investment. A competing firm can attract all of these workers by decreasing the spread because investment costs fall by more than

expected product, raising expected net earnings. Increasing marginal cost of skill implies a unique equilibrium spread between the prizes that maximizes expected utility.

More precisely, consider the contestant's problem, assuming that both have the same costs of investment $C(\mu)$, so that their behavior is identical. A contestant's expected utility (wealth) is

$$(P)[W_1 - C(\mu)] + (1 - P)[W_2 - C(\mu)] = PW_1 + (1 - P)W_2 - C(\mu), \tag{2}$$

where P is the probability of winning. The probability that j wins is

$$\begin{aligned} P &= \text{prob}\,(q_j > q_k) = \text{prob}\,(\mu_j - \mu_k > \epsilon_k - \epsilon_j) \\ &= \text{prob}\,(\mu_j - \mu_k > \xi) = G(\mu_j - \mu_k), \end{aligned} \tag{3}$$

where $\xi \equiv \epsilon_k - \epsilon_j$, $\xi \sim g(\xi)$, $G(\cdot)$ is the cdf of ξ, $E(\xi) = 0$, and $E(\xi^2) = 2\sigma^2$ (because ϵ_j and ϵ_k are i.i.d.). Each player chooses μ_i to maximize (2). Assuming interior solutions, this implies

$$(W_1 - W_2)\frac{\partial P}{\partial \mu_i} - C'(\mu_i) = 0$$

and

$$(W_1 - W_2)\frac{\partial^2 P}{\partial \mu_i^2} - C''(\mu_i) < 0, i = j, k. \tag{4}$$

We adopt the Nash-Cournot assumptions that each player optimizes against the optimum investment of his opponent, since he plays against the market over which he has no influence. Therefore, j takes μ_k as given in determining his investment and conversely for k. It then follows from (3) that, for player j

$$\partial P/\partial \mu_j = \partial G(\mu_j - \mu_k)/\partial \mu_j = g(\mu_j - \mu_k),$$

which upon substitution into (4) yields j's reaction function

$$(W_1 - W_2)g(\mu_j - \mu_k) - C'(\mu_j) = 0. \tag{5}$$

Player k's reaction function is symmetrical with (5).

Symmetry implies that when the Nash solution exists, $\mu_j = \mu_k$ and $P = G(0) = \frac{1}{2}$, so the outcome is purely random in equilibrium. Ex ante, each player affects his probability of winning by investing.[2]

[2] However, it is not necessarily true that there is a solution because with arbitrary density functions the objective function may not be concave in the relevant range. It is possible to show that a pure strategy solution exists provided that σ^2 is sufficiently large: Contests are feasible only when chance is a significant factor. This result accords with intuition and is in the spirit of the old saying that a (sufficient) difference of opinion is necessary for a horse race. Stated otherwise, since $\partial P/\partial \mu_j = g(\mu_j - \mu_k)$ and $g(\cdot)$ is a pdf, $\partial^2 P/\partial \mu_j^2 = g'(\mu_j - \mu_k)$ may be positive, and fulfillment of second-order conditions in (4) implies sharp breaks in the reaction function. If σ^2 is small enough the breaks occur at very low levels of investment, and a Nash equilibrium in pure strategies will not exist. Existence of an equilibrium is assumed in all that follows.

Substituting $\mu_j = \mu_k$ at the Nash equilibrium, equation (5) reduces to

$$C'(\mu_i) = (W_1 - W_2)g(0), \quad i = j, k, \tag{6}$$

verifying the point above that players' investments depend on the spread between winning and losing prizes. Levels of the prizes only influence the decision to enter the game, which requires nonnegativity of expected wealth.

The risk-neutral firm's realized gross receipts are $(q_j + q_k) \cdot V$, and its costs are the total prize money offered, $W_1 + W_2$. Competition for labor bids up the purse to the point where expected total receipts equal costs $W_1 + W_2 = (\mu_j + \mu_k) \cdot V$. But since $\mu_j = \mu_k = \mu$ in equilibrium, the zero-profit condition reduces to

$$V\mu = (W_1 + W_2)/2. \tag{7}$$

The expected value of product equals the expected prize in equilibrium. Substitute (7) into the worker's utility function (2). Noting that $P = \frac{1}{2}$ in equilibrium, the worker's expected utility at the optimum investment strategy is

$$V\mu - C(\mu). \tag{8}$$

The equilibrium prize structure selects W_1 and W_2 to maximize (8), or

$$[V - C'(\mu)](\partial\mu/\partial W_i) = 0, \quad i = 1, 2. \tag{9}$$

The marginal cost of investment equals its marginal social return, $V = C'(\mu)$, in the tournament as well as the piece rate. Therefore, competitive tournaments, like piece rates, are efficient and both result in exactly the same allocation of resources.

Some further manipulation of the equilibrium conditions yields an interesting interpretation in terms of the theory of agency (see Ross 1973; Becker and Stigler 1974; Harris and Raviv 1978; and Lazear 1979):

$$\begin{aligned} W_1 &= V\mu + C'(\mu)/2g(0) = V\mu + V/2g(0) \\ W_2 &= V\mu - C'(\mu)/2g(0) = V\mu - V/2g(0). \end{aligned} \tag{10}$$

The second equality follows from $V = C'(\mu)$. Now think of the term $C'(\mu)/2g(0) = V/2g(0)$ in (10) as an entrance fee or bond that is posted by each player. The winning and losing prizes pay off the expected marginal value product plus or minus the entrance fee. That is, the players receive their expected product combined with a fair winner-take-all gamble over the total entrance fees or bonds. The appropriate social investment incentives are given by each contestant's attempt to win the gamble. This contrasts with the main agency result, where the bond is returned to each worker after a satisfactory performance has

been observed. There the incentive mechanism works through the employee's attempts to work hard enough to recoup his own bond. Here it works through the attempts to win the gamble.

Comparative statics for this problem all follow from (9) and (10) once a distribution is specified. For example, if ϵ is normal with variance σ^2, then $g(0) = \frac{1}{2}\sigma\sqrt{\pi}$. It follows from (10) that the optimal spread varies directly with V and σ^2. While several other interesting observations can be made of this sort, we note a somewhat different but important practical implication of this general scheme. Even though the optimal prize structure determines expected marginal product through its effect on worker choice of μ and the zero-profit condition (7) implies that expected prizes equal expected productivity, nevertheless actual realized earnings definitely do not equal productivity in either an ex ante or ex post sense. Consider ex ante first. Since $\mu_j = \mu_k = \mu$, expected products are equal. Since $W_1 > W_2$ is required to induce any investment, the payment that j receives never equals the payment that k receives. It is impossible that the prize is equal to ex ante product, because ex ante products are equal. Nor do wages equal ex post products. Actual product is Vq rather than $V\mu$. But q is a random variable, the value of which is not known until after the game is played, while W_1 and W_2 are fixed in advance. Only under the rarest coincidence would $W_1 = Vq_j$ and $W_2 = Vq_k$.

Consider the salary structure for executives. It appears as though the salary of, say, the vice-president of a particular corporation is substantially below that of the president of the same corporation. Yet presidents are often chosen from the ranks of vice-presidents. On the day that a given individual is promoted from vice-president to president, his salary may triple. It is difficult to argue that his skills have tripled in that 1-day period, presenting difficulties for standard theory where supply factors should keep wages in those two occupations approximately equal. It is not a puzzle, however, when interpreted in the context of a prize. The president of a corporation is viewed as the winner of a contest in which he receives the higher prize, W_1. His wage is settled on not necessarily because it reflects his current productivity as president, but rather because it induces that individual and all other individuals to perform appropriately when they are in more junior positions. This interpretation suggests that presidents of large corporations do not necessarily earn high wages because they are more productive as presidents but because this particular type of payment structure makes them more productive over their entire working lives. A contest provides the proper incentives for skill acquisition prior to coming into the position.[3]

[3] If ϵ is a fixed effect, there is additional information from knowing the identity of winners and losers. The expected productivity of a winner is $\mu + E(\epsilon_j \mid q_j > q_k)$, while that of a loser is $\mu + E(\epsilon_j \mid q_j < q_k)$. In a one-period contest there is no possibility of

Comparisons

Though tournaments and piece rates are substantially different institutions for creating incentives, we have demonstrated the surprising result that both achieve the Pareto optimal allocation of resources when workers are risk neutral. In fact other schemes also achieve this allocation. For example, instead of playing against an opponent, a worker might be compared with a fixed standard $\bar{q}$, with one payment awarded if output falls anywhere below $\bar{q}$ and another, higher, payment awarded if output falls anywhere above standard. Attempting to beat the standard has the same incentive effects as attempting to beat another player. Using the same methods as above, it is not difficult to show that there are spread-standard combinations that induce Pareto optimum investments. Since all these schemes involve the same investment policy, and since average payout by the firm equals average product for all of them, they all yield the same expected rewards and, therefore, the same expected utility to workers.[4]

In spite of the apparent equality of these schemes in terms of the preferences of risk-neutral workers, considerations of differential costs of information and measurement may serve to break these ties in practical situations. The essential point follows from the theory of measurement (Stevens 1968) that a cardinal scale is based on an underlying ordering of objects or an ordinal scale. In that sense, an ordinal scale is "weaker" and has fewer requirements than a cardinal scale. If it is less costly to observe rank than an individual's level of output, then tournaments dominate piece rates and standards. On the other hand, occupations for which output is easily observed save resources by using the piece rate or standard, or some combination, and avoid the necessity of making direct comparisons with others as the tournament requires. Salesmen, whose output level is easily observed, typically are paid by piece rates, whereas corporate executives, whose output is more difficult to observe, engage in contests.

In a modern, complex business organization, a person's productivity as chief executive officer is measured by his effect on the profitability of the whole enterprise. Yet the costs of measurement for

taking advantage of this information. However, in a sequential contest with no firm-specific capital, the information would be valuable and would constrain subsequent wage payments in successive rounds through competition from other firms. It is not difficult to show that this does not affect the general nature of the bond-gamble solution. Alternatively, if the investment has firm-specific elements or firms adopt policies that bind workers to it (as in Lazear 1979), these restrictions do not necessarily apply.

[4] The level of the standard is indeterminate, since for any $\bar{q}$ a corresponding spread can be chosen to achieve the optimal investment. This is also true of contests among more than two players. With N contestants, the prizes of $N - 2$ of them are indeterminate. When risk neutrality is dropped, the indeterminacy vanishes in both cases.

each conceivable candidate are prohibitively expensive. Instead, it might be said that those in the running are "tested" by assessments of performance at lower positions. Realizations from such tests are sample statistics in these assessments, in much the same way that grades are assigned in a college classroom and IQ scores are determined. The point is that such tests are inherently ordinal in nature, even though the profitability of the enterprise is metered by a well-defined, cardinal ratio scale. It is in situations such as this that the conditions seem ripe for tournaments to be the dominant incentive contract institution.

Notice in this connection that the basic prize and piece-rate structures survive a broad class of revenue functions other than summable ones. Even if the production function of the firm includes complicated interactions involving complementarity or substitution among individual outputs, there exists the possibility of paying workers either on the basis of individual performance or by rank order. The revenue function itself can even involve rank-order considerations, and both possibilities still exist. For example, spectators at a horse race generally are interested in the speed of the winning horse and the closeness of the contest. Then the firm's (track) revenue function depends on the first few order statistics; yet the horses could be paid on the basis of their speed rather than on the basis of win, place, and show positions. Both methods would induce them to run fast.[5]

There has been very little treatment of the problem of tournament prize structure and incentives in the literature. Little else but the well-known paper by Friedman (1953) based on Friedman-Savage preferences for lotteries exists in economics. In the statistics literature there is an early paper by Galton (1902) that is worthy of brief discussion. Galton inquired into the ratio of first- and second-place prize money in a race of n contestants, assuming the prizes were divided in the following ratio:

$$W_1/W_2 = (Q_1 - Q_3)/(Q_2 - Q_3).$$

Here Q_1 is the expected value of the first- (fastest) order statistic, etc. While a moment's reflection suggests this criterion to be roughly related to marginal productivity, Galton proposed it on strictly a priori grounds. He went on to show the remarkable result that the ratio above

[5] The reader is reminded that throughout this section and the next workers are identical a priori and differ only ex post through the realization of ϵ. In the real world, where there is population heterogeneity, market participants are sorted into different contests. There players (and horses, for that matter) who are known to be of higher quality ex ante may play in games with higher stakes. If it can be accomplished, the sorting is by anticipated marginal products. In that sense, pay differences among contestants of known quality resemble the effect of a "piece rate." These issues are more thoroughly discussed below.

is approximately 3 when the parent distribution of speed is normal. Hence, this criterion results in a highly skewed prize structure. From what we know today about the characteristic skew of extreme value distributions, a skewed reward structure based on order statistics is less surprising for virtually any parent distribution. In the more modern statistical literature, the method of paired comparisons has tournament-like features. Samples from different populations are compared pairwise, and the object is to choose the one with the largest mean. Comparing all samples to each other is like a round-robin tournament. An alternative design is a knockout tournament with single or double elimination. The latter requires fewer samples and is therefore cheaper, but does not generate as much information as the round robin (David 1963; Gibbons, Olkin, and Sobel 1977).

Galton's original work and the more modern developments it has given rise to are not helpful to us; they deal with samples from fixed populations, so the reward structure is irrelevant for resource allocation. The problem we have treated here is that of choosing the reward structure to provide the proper incentive and elicit the socially proper distributions.

III. Optimal Compensation with Risk Aversion

All compensation systems can be viewed as schemes which transform the distribution of productivity to a distribution of earnings. A piece rate is a linear transformation of output, so the distribution of income is the same apart from a change in location and scale. A tournament is a highly nonlinear transformation: It converts the continuous distribution of productivity into a discrete, binomial distribution of income. When workers are risk neutral, both schemes yield identical investments and expected utility because their first moments are the same. In this section, it is shown that with risk aversion one method or the other usually yields higher expected utility, because the interaction between insurance and action implies substantially different first and second moments of the income distribution in the two cases.[6]

We have been unable to completely characterize the conditions under which piece rates dominate rank-order tournaments and vice versa, but we show some examples here. Truncation offered by prizes implies more control of extreme values than piece rates but less control of the middle of the distribution. Different utility functions

[6] One might think that risks could be pooled among groups of workers through sharing agreements, but that is false because of moral hazard. A worker would never agree to share prizes since doing so would result in $\mu = 0$, and consequently $E(q_j + q_k) = 0$ and bankruptcy for the firm. As a result, firms offering tournaments or piece rates in the pure sense yield higher expected utility than the sharing arrangement.

weight one aspect more than the other so that tournaments can actually dominate piece rates.

Optimum Linear Piece Rate[7]

The piece-rate scheme analyzed pays workers a guarantee, I, plus an incentive, rq, where r is the piece rate per unit of output. The problem for the firm is to pick an r, I combination that maximizes workers' expected utility

$$\max_{I,r} [E(U) = \max \int U(y)\theta(y)dy], \tag{11}$$

where

$$\begin{aligned} y &\equiv I + rq - C(\mu) \\ &= I + r\mu + r\epsilon - C(\mu) \end{aligned} \tag{12}$$

and $\theta(y)$ is the pdf of y.

The worker's problem is to choose μ to maximize expected utility given I and r. If $\epsilon \sim f(\epsilon)$, the worker's problem is

$$\max_{\mu} E(U) = \int U[I + r\mu + r\epsilon - C(\mu)]f(\epsilon)d\epsilon.$$

The first-order condition is

$$\frac{\partial E(U)}{\partial \mu} = \int [U'(y)][r - C'(\mu)]f(\epsilon)d\epsilon = 0,$$

which conveniently factors so that

$$r = C'(\mu). \tag{13}$$

Condition (13) is identical to the risk-neutral case, because ϵ is independent of investment effort, μ.

Assuming risk-neutral employers, $V\mu$ is expected revenue from a worker and $I + r\mu$ is expected wage payments. Therefore, the zero-profit market constraint is

$$V\mu = I + r\mu. \tag{14}$$

Solving (14) for I and substituting into (12), the optimum contract maximizes

$$\int U\{V\mu(r) + r\epsilon - C[\mu(r)]\}f(\epsilon)d\epsilon$$

with respect to r, where $\mu = \mu(r)$ satisfies (13). After simplification the

[7] The following is similar to a problem analyzed by Stiglitz (1975). A linear piece-rate structure is a simplification. A more general structure would allow for nonlinear piece rates (see Mirrlees 1976).

marginal condition is

$$[V - C'(\mu)]\frac{d\mu}{dr}EU' + E\epsilon U' = 0. \tag{15}$$

Since risk aversion implies $E\epsilon U' < 0$, (15) shows that $V > C'(\mu)$ in the optimum contract for risk-averse workers. This underinvestment is the moral hazard resulting from insurance $I > 0$ and $r < V$ implied by (15).

Using familiar Taylor series approximations to the utility function and a normal density for ϵ, the optimum is approximated by

$$\mu \doteq C'^{-1}\left(\frac{V}{1 + sC''\sigma^2}\right) \tag{16}$$

and

$$\sigma_y^2 \doteq \frac{V^2\sigma^2}{(1 + sC''\sigma^2)^2}, \tag{17}$$

where $s \equiv -U''/U'$ evaluated at mean income is the measure of absolute risk aversion. Investment increases (see [16]) in V and decreases in s, C'', and σ^2, because all these changes imply similar changes in the marginal piece rate r which influences investment through condition (13). The same changes in V, s, and C'' have corresponding effects on the variance of income (see [17]), but an increase in σ^2 actually reduces variance, if σ^2 is large, because it reduces r and increases I.[8]

Optimum Prize Structure

The worker's expected utility in a two-player game is

$$E(U) = P\{U[W_1 - C(\mu^*)]\} + (1 - P)\{U[W_2 - C(\mu^*)]\}, \tag{18}$$

where * denotes the outcome of the contest rather than the piece-rate scheme. The optimum prize structure is the solution to

$$\max_{W_1,W_2}\left(E(U^*) = \max_{\mu^*}\{P \cdot U[W_1 - C(\mu^*)] + (1 - P) \cdot U[W_2 - C(\mu^*)]\}\right) \tag{19}$$

subject to the zero-profit constraint

$$V\mu^* = PW_1 + (1 - P)W_2. \tag{20}$$

The worker selects μ^* to satisfy $\partial E(U)/\partial\mu^* = 0$. Since cost functions are the same and ϵ_j and ϵ_k are i.i.d., the Nash solution implies $\mu_j = \mu_k$

[8] Furthermore, $r \doteq V/(1 + sC''\sigma^2)$ and $I \doteq sV^2\sigma^2/(1 + sC''\sigma^2)^2$, so that $r = V$ and $I = 0$ in the case of risk neutrality ($s = 0$). All these approximations use first-order expansions for terms in $U'(\cdot)$ and second-order expansions for terms in $U(\cdot)$. The same is true of the approximations below for the tournament.

and $P = \frac{1}{2}$ as before. Then the worker's investment behavior simplifies to

$$C'(\mu^*) = \frac{2[U(1) - U(2)]g(0)}{U'(1) + U'(2)}, \tag{21}$$

where $U(\tau) \equiv U[W_\tau - C(\mu^*)]$ and $U'(\tau) \equiv U'[W_\tau - C(\mu^*)]$ for $\tau = 1, 2$. Equation (21) implies

$$\mu^* = \mu^*(W_1, W_2), \tag{22}$$

and the optimum contract (W_1, W_2) maximizes

$$E(U^*) = \tfrac{1}{2} U[W_1 - C(\mu^*)] + \tfrac{1}{2} U[W_2 - C(\mu^*)] \tag{23}$$

subject to (20), with $P = \frac{1}{2}$, and (22). Increasing marginal cost of investment and risk aversion guarantees a unique maximum to (23) when a Nash solution exists. Again, assuming a normal density for ϵ, second-order approximations yield

$$\mu^* \doteq C'^{-1}\left(\frac{V}{1 + sC''\sigma^2\pi}\right) \tag{24}$$

and

$$\sigma_{y^*}^2 \doteq \frac{\pi V^2 \sigma^2}{(1 + \pi s C''\sigma^2)^2}, \tag{25}$$

where

$$y^* \begin{cases} = W_1 - C(\mu^*) \text{ if } q_j > q_k \\ = W_2 - C(\mu^*) \text{ if } q_j < q_k \end{cases}$$

and $\epsilon_j \sim N(0,\sigma^2), \epsilon_k \sim N(0,\sigma^2)$, and cov $(\epsilon_j, \epsilon_k) = 0$.[9] The comparative statics of (24) and (25) are similar to the piece rate (16) and (17) and need not be repeated.

Comparisons

Equations (16) and (24) indicate that investment and expected income[10] are lower for the contest than for the piece rate at given values of s. Moreover, for values of σ^2 in excess of $1/sC''\sqrt{\pi}$, the variance of income in the tournament is smaller than for the piece rate. This would seem to suggest that contests provide a crude form of insurance when the variance of chance is large enough, but the problem is significantly more complicated than that because there is no separation between tastes and opportunities in this problem: The optimum

[9] Futhermore, $C'(\mu^*) \doteq g(0)(W_1 - W_2)$, so the spread is still crucial for investment incentives, as in the risk-neutral case.

[10] Since $y = V\mu - C(\mu)$, and since μ is below the wealth-maximizing level of μ when workers are risk averse, lower μ implies lower y because revenue falls by more than cost.

mean and variance themselves depend on utility-function parameters. Thus, for example, for the constant, absolute risk-aversion utility function $U = -e^{-sy}/s$, the insurance provided by the contest is insufficient to compensate for its smaller mean: It can be shown that the expected indirect utility of the optimal piece rate exceeds that of the optimal tournament for all values of σ^2, at least with normal distributions and quadratic investment-cost functions. However, when there is declining absolute risk aversion, we have examples where the contest dominates the piece rate.

Illustrative calculations are shown in table 1 using the utility function $U = \alpha y^{\alpha}$, which exhibits constant relative but declining absolute risk aversion, $s(y) = (1 - \alpha)/y$. Again quadratic costs and normal errors are assumed. However, this utility function is defined for positive incomes only, so an amount of nonlabor income y_0 is assigned to the worker to avoid a major approximation error of the normal, which admits negative incomes (i.e., the possibility of losses).

Table 1 shows that when $y_0 = 100$ so that $s = .005$, the contest is preferred until $\sigma^2 \geq 3$. However, if $y_0 = 25$ so that $s = .020$, the contest is only preferred for $\sigma^2 < .2$. The intuition is that piece rates concentrate the mass of the income distribution near the mean, while contests place 50 percent of the weight at one value significantly below the mean and the other value significantly above. Strongly risk-averse workers seem to dislike the binomial nature of this distribution when σ^2 is high because it concentrates too much of the mass at low levels of utility. However, when σ^2 is small, the contest which truncates the tails

TABLE 1

CONSTANT RELATIVE RISK AVERSION

σ^2	μ	μ^*	$E(U)$	$E(U^*)$
		$y_0 = 100; s(y_0) = .005$		
.1	.9995	.9984	5.012155	5.012465
.5	.9975	.9922	5.012150	5.012445
1	.9950	.9846	5.012100	5.012295
3	.9852	.9552	5.011940	5.011925
6	.9710	.9142	5.011800	5.011415
12	.9436	.8420	5.011420	5.010515
		$y_0 = 25; s(y_0) = .020$		
.1	.9980	.9938	2.524665	2.524725
.2	.9960	.9878	2.524616	2.524575
1	.9807	.9419	2.524237	2.523437
12	.8094	.5741	2.519930	2.514282

NOTE.—$U = \alpha y^{\alpha}$; $y = y_0 + I + rq - C(\mu)$ for piece rate; $y = y_0 + W_i - C(\mu)$ for contest ($i = 1, 2$); $\alpha = .5, V = 1, C(\mu) = \mu^2/2$; σ^2.

of the income distribution associated with a linear piece rate has higher value.

Income Distributions

While it is not possible to make a general argument based on an example, table 1 suggests that persons with more endowed income and smaller absolute risk aversion are more likely to prefer contests, and those with low levels of endowed wealth and larger absolute risk aversion are more likely to prefer piece rates. Consider a situation in which all persons have the same utility function, such as the one in table 1, and face the same costs and luck distribution, the only difference being the fact that some workers have larger endowed incomes than others. If this difference is large enough, it can be optimal to pay piece rates to those with small values of endowed income and to pay prizes to those with large values. Individuals will self-select the payment scheme in accordance with their wealth. The distribution of earnings among those selecting the piece-rate jobs is normal with mean $V\mu$ and variance $r^2\sigma^2$. It is binomial with mean $V\mu^*$ and variance $(\Delta W)^2/4$ for those who enter tournaments. Note that μ and μ^* depend upon $s(y)$, which is smaller for workers who select contests, and it can turn out as it does in table 1 that expected income is larger in the contest than in the piece rate; for example, if $\sigma^2 = 1$ then the rich prefer contests ($5.012295 > 5.012100$) and the poor prefer piece rates ($2.524237 > 2.523437$), but $\mu^* = .9846$ exceeds $\mu = .9807$. This situation is shown in figure 1.

The overall distribution is the sum of a binomial and a normal with lower mean, weighted by the number of individuals in each occupation (see fig. 1). It is positively skewed because $V\mu^* > V\mu$. Note also that the distribution of wage income will be less skewed than that of total income. The reason is that y_0 and mean-wage income are positively correlated because the likelihood of choosing a contest increases with y_0. These implications conform to the standard findings on the distribution of income in an economy.

This example is interesting because it is very closely related to some early results of Friedman (1953), who studied how alternative social arrangements can produce income distributions that cater to workers' risk preferences. He showed that the Friedman-Savage utility function leads to a two-class distribution. Persons in the risk-averse region are assigned to occupations in which income follows productivity, while persons in the risk-preferring region buy lottery tickets in very risky occupations in which few win very large prizes. The overall distribution is the sum of these two and exhibits characteristic skew. The Friedman-Savage utility function implies that a person's risk

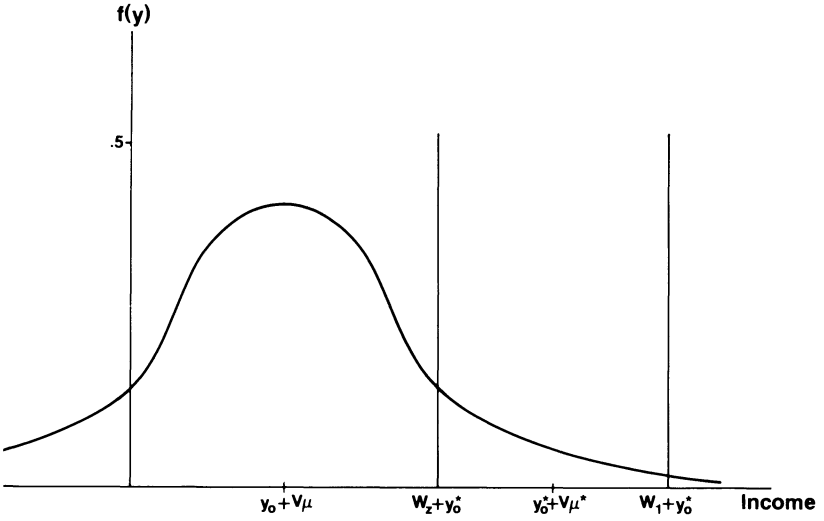

FIG. 1

preferences depend on the part of his wealth that is not at risk. Therefore, Friedman's assignment of people to jobs really follows endowed wealth (y_0), just as in our example. However, our framework offers two improvements. First, the problem of incentives is directly incorporated into the formulation of the optimum policy. Second, workers in this model are risk averse for all values of incomes, but even so gambles can be the optimal policy.

Error Structure

Relative costs of measurement are still important in choosing among incentive schemes, but the error structure plays additional roles when workers are risk averse. Suppose the output estimator for worker i in activity τ is $\hat{q}_{i\tau} = q_{i\tau} + \rho_\tau + \nu_{i\tau}$, where $\nu_{i\tau}$ is random error and ρ_τ is an error that is specific to activity τ but common to all workers within that activity. In the piece rate the common error ρ adds noise which risk-averse workers dislike, while the common noise drops out of a rank-order comparison because it affects both contestants similarly. That is, the relevant variance for the contest is $2\sigma_\nu^2$, while that for the piece rate is $\sigma_\rho^2 + \sigma_\nu^2$. It is evident that this can tip the balance in favor of tournaments if σ_ρ^2 is large enough and/or workers are sufficiently risk averse.

The common error ρ bears two interesting interpretations. One is activity-specific measurement error. For example, j and k may have the same supervisor whose biased assessments affect all workers similarly. This is similar to monitoring all workers by a mechanical count-

ing device that might run too fast or too slow in any given trial. The other interpretation of ρ is true random variation that affects the enterprise as a whole. For example, suppose all firms produce with the same technology, but that in a given period some firms do better or worse than others. Then risk-averse workers prefer not to have their incomes vary with conditions facing the firm as a whole, and wages based on a contest eliminate this kind of variation. Without its elimination there would be excessive losses due to moral hazard.

It must be pointed out that, in the absence of measurement error, using a contest against a fixed standard $\bar{q}$ discussed above has lower variance than playing against an opponent. As shown in Section II, the relevant variance in a contest is that of $\xi = \epsilon_k - \epsilon_j$, which has variance $2\sigma^2$ against an opponent and only σ^2 against a standard (since the standard is invariant, $\epsilon_k \equiv 0$). Consequently, we might expect risk-averse workers to prefer absolute standards.[11] Again, however, the crucial issue is the costs of measurement and the error structure. For the complex attributes required for managerial positions, it is difficult to observe output and therefore difficult to compare to an absolute standard. Insofar as samples and tests are necessary, it bears repeating that these are inherently ordinal in nature. But this leads us back to the problem of common error, where it is often impossible to know whether a person's output is satisfactory without comparisons to other persons. Further, when there are changing production circumstances in the firm as a whole, it is difficult to know whether the person failed to meet the standard because of insufficient investment or because the firm was generally experiencing bad times, a problem of measuring "value added." Risk-averse workers increase utility by competing against an opponent and eliminating this kind of firm effect.

IV. Heterogeneous Contestants

Workers are not sprinkled randomly among firms but rather seem to be sorted by ability levels. One explanation for this has to do with complementarities in production. But even in the absence of complementarities, sorting may be an integral part of optimal labor-contract arrangements. Informational considerations imply that

[11] Playing against a standard is like Mirrlees's (1976) notion of an "instruction." It is clear that using standards as well as piece rates must be superior to using one alone. That scheme would allow workers to be paid $\bar{I}$ if $q < \bar{q}$ and $I_0 + rq$ for $q \geq \bar{q}$. This is important because it truncates the possibilities when $Vq < 0$. Given the technology, it is possible that very large negative values of output can occur, and since it is impossible to always tax workers the full extent of this loss, some form of truncation is desirable. A contest is an alternative way to control the tails of this distribution.

compensation methods may affect the allocation of worker types to firms. Therefore, this section returns to the case of risk neutrality and analyzes tournament structures when investment costs differ among persons. Two types of persons are assumed, a's and b's, with marginal costs of the a's being smaller than those of the b's: $C_a'(\mu) < C_b'(\mu)$ for all μ. The distribution of disturbances $f(\epsilon)$ is assumed to be the same for both groups. Many of the following results continue to hold, with usually obvious modification of the arguments, if the a's and b's draw from different distributions. The following section addresses the question of self-selection when workers know their identities but firms do not. The next section discusses handicapping schemes when all cost-function differences can be observed by all parties.

Adverse Selection

Suppose that each person knows to which class he belongs but that this information is not available to anyone else. The principal result is that the a's and b's do not self-sort into their own "leagues." Instead, all workers prefer to work in firms with the best workers (the major leagues). Furthermore, there is no pure price-rationing mechanism that induces Pareto optimal self-selection. But mixed play is inefficient because it cannot sustain the proper investment strategies. Therefore, tournament structures naturally require credentials and other nonprice signals to differentiate people and assign them to the appropriate contest. Firms select their employees based on such information as past performances, and some are not permitted to compete.

The proof of adverse selection consists of two parts. First we show that players do not self-sort into a leagues and b leagues. Second, we show that the resulting mixed leagues are inefficient.

1. *Players do not self-sort.*—Assume leagues are separated and consider the expected revenue R_i generated by playing in league $i = a, b$ with an arbitrary investment level μ. Then

$$R_i(\mu) = W_2^i + (W_1^i - W_2^i)P^i, \quad i = a, b, \tag{26}$$

where (W_1^i, W_2^i) is the prize money, and P^i is the probability of winning in league i. Recall that P^i depends on the individual's level of investment and that of his rivals. Therefore, $P^a = G(\mu - \mu_a^*)$ and $P^b = G(\mu - \mu_b^*)$, where μ_a^* is the existing players' investments in the a league, where $V = C_a'(\mu_a^*)$, and similarly for μ_b^*. Recalling from (6) and (9) that $W_1^i - W_2^i = V/g(0)$ and from (10) that $W_2^i = V\mu_i^* - V/2g(0)$, equation (26) becomes

$$R_i(\mu) = V\mu_i^* - \frac{V}{g(0)}[\tfrac{1}{2} - G(\mu - \mu_i^*)]. \tag{27}$$

Note that $R_i(\mu) = V\mu_i^*$ when $\mu = \mu_i^*$ and that $dR_i/d\mu \equiv R_i'(\mu) = Vg(\mu - \mu_i^*)/g(0) > 0$. Since $C_a'(\mu_a^*) = V$ and $C_b'(\mu_b^*) = V$, then $\mu_b^* < \mu_a^*$ so that $R_i(\mu_a^*) > R_i(\mu_b^*)$. Furthermore, $R_b'[\mu - (\mu_a^* - \mu_b^*)] = R_a'(\mu)$. Therefore, $R_b(\mu)$ is a pure displacement of $R_a(\mu)$. Since $R_i'(\mu) = V$ for $\mu = \mu_i^*$ and $R_i'(\mu) < V$ elsewhere, and since $R_i(\mu)$ is increasing, the revenue functions never cross. So $R_b(\mu)$ lies to the southwest of $R_a(\mu)$ (see fig. 2). Therefore, independent of cost curves, it is always better to play in the a league than the b league: Workers will not self-select.

2. *Mixed contests are inefficient.*—Suppose the proportions of a's and b's in the population are α and $(1 - \alpha)$, respectively. If pairings among a's and b's are random, then expected utility of a player of type i is

$$\overline{W}_2 + [\alpha P_a^i + (1 - \alpha)P_b^i](W_1 - W_2) - C_i(\mu_i),$$

where $(\overline{W}_1, \overline{W}_2)$ is the prize money in mixed play and P_j^i is the probability that a player of type i defeats a player of type j. The first-order condition for investment of type i in this game is

$$\left[\alpha \frac{\partial P_a^i}{\partial \mu_i} + (1 - \alpha)\frac{\partial P_b^i}{\partial \mu_i}\right] \cdot (\overline{W}_1 - \overline{W}_2) = C_i'(\overline{\mu}_i).$$

A development similar to Section II implies equilibrium reaction functions

$$[\alpha g(0) + (1 - \alpha)g(\overline{\mu}_a - \overline{\mu}_b)](W_1 - W_2) = C_a'(\overline{\mu}_a)$$

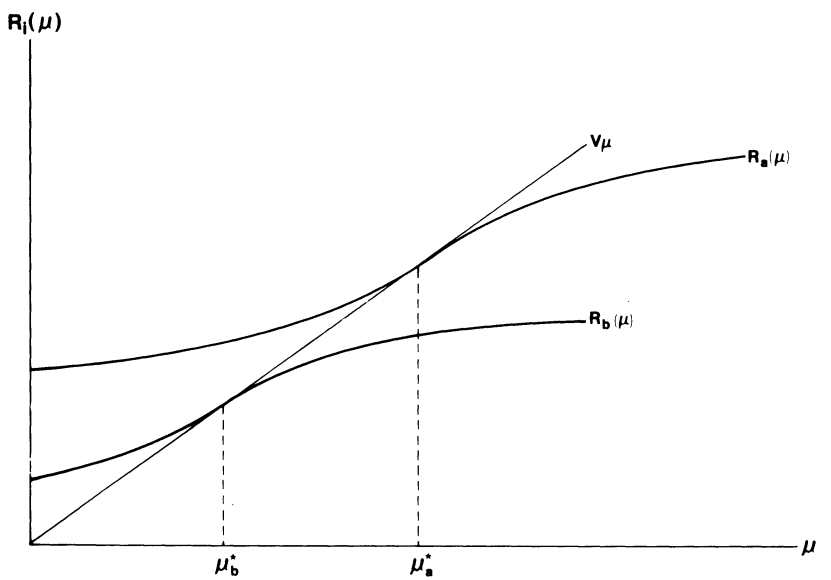

FIG. 2

for a's and

$$[\alpha g(\bar{\mu}_b - \bar{\mu}_a) + (1 - \alpha)g(0)](W_1 - W_2) = C_b'(\bar{\mu}_b)$$

for b's. If the solution is efficient, then $C_b'(\mu_b) = V = C_a'(\mu_a)$, which implies

$$\alpha g(0) + (1 - \alpha)g(\bar{\mu}_a - \bar{\mu}_b) = \alpha g(\bar{\mu}_b - \bar{\mu}_a) + (1 - \alpha)g(0).$$

Since g is symmetric and nonuniform, this condition can hold only if $\alpha = \frac{1}{2}$. Therefore, except in that very special case, mixed contests yield inefficient investment: One type of player overinvests and the other underinvests depending upon whether or not $\alpha \gtrless \frac{1}{2}$.

We conclude that a pure price system cannot sustain an efficient competitive equilibrium in the presence of population heterogeneity with asymmetric information. Markets can be separated, but only at a cost. Consider, for example, the case where a's want to prevent b's from contaminating their league. By making the spread, $W_1^a - W_2^a$, sufficiently large, $R_a(\mu)$ becomes steeper than $R_b(\mu)$ in figure 2 and crosses it so that the envelope covers $R_b(\mu)$ at low values of μ and $R_a(\mu)$ at high values. Then, for some high levels of μ, it is more profitable to play in the a league and, for low levels of μ, the b league is preferable. Individuals may self-sort, but the cost is that a's overinvest. The result is akin to that of Akerlof (1976) and to those of Spence (1973), Riley (1975), Rothschild and Stiglitz (1976), and Wilson (1977). As they show, a separating equilibrium need not exist, but, even if it does, that equilibrium may be inferior to a nonseparating equilibrium.

The obvious practical resolution of these difficulties is the use of nonprice rationing and certification to sort people into the appropriate leagues based on past performance. Similarly, firms use nonprice factors to allocate jobs among applicants. The rules for allocating those jobs may be important for at least two reasons that we can only briefly describe here.

First, sorting workers of different skill levels into appropriate positions within a hierarchy may be beneficial. In this paper, production is additive, so it does not matter who works with whom. To the extent that the production technology is somewhat more complicated, sorting may well be crucial. A series of pairwise, sequential contests may efficiently perform that function. Suppose that $q_{it} = \mu_i + \delta_i + \eta_{it}$, where δ_i is an unobserved ability component for player i and η is white noise. Suppose it is efficient for the individual with the highest δ to be the chief executive. There will be a tendency to have winners play winners because

$$E(\delta_j \mid q_{j1} > q_{k1}) > E(\delta_k \mid q_{j1} > q_{k1})$$

in the first round. A sequential elimination tournament may be a cost-efficient way to select the best person.

Second, workers may not know precisely their own abilities or cost functions. A worker who is ignorant about his cost function values information before selecting a level of investment expenditure. Therefore, firms may offer "tryouts" to provide information about optimal investment strategies. In fact, one can imagine the existence of firms which specialize in running contests among young workers—the minor leagues—which provide information to be used when and if the workers opt to increase the stakes and enter a bigger league.

These issues point up an important difference between piece rates and contests. In the pure heterogeneous case, where information is asymmetric and workers are risk neutral, a piece rate always yields an efficient solution, namely, $V = C_a'(\mu_a) = C_b'(\mu_b)$. However, once slotting of workers is important because of complementarities in production, or if it is desirable for workers to gain information about their type, it is no longer obvious that a series of sequential contests does not result in a superior allocation of resources.

Handicap Systems

This section moves to the opposite extreme of the previous discussion and assumes that the identities of each type of player are known to everyone. Competitive handicaps yield efficient mixed contests.

Consider again two types a and b now known to everyone. Prize structures in a-a and b-b tournaments satisfying (11) and (12) are efficient, but those conditions are not optimal in mixed a-b play. Denote the socially optimal levels of investment by μ_a^* and μ_b^*, their difference by $\Delta\mu$, and the prizes in a mixed league by $\tilde{W}_1$ and $\tilde{W}_2$. Let h be the handicap awarded to the inferior player b. Then the Nash solution in the a-b tournament satisfies

$$g(\mu_a - \mu_b - h)\Delta\tilde{W} = C_a'(\mu_a) \tag{28}$$

and

$$g(\mu_a - \mu_b - h)\Delta\tilde{W} = C_b'(\mu_b).$$

(The second condition in [28] follows from symmetry of $g[\xi]$.) Since the efficient investment criterion is $V = C_a'(\mu_a^*) = C_b'(\mu_b^*)$, independent of pairings, the optimum spread in a mixed match must be

$$\Delta\tilde{W} = V/g(\Delta\mu - h). \tag{29}$$

From (28), condition (29) insures the proper investments by both contestants. The spread is larger in mixed than pure contests unless a

gives b the full handicap $h = \mu_a^* - \mu_b^*$. Otherwise, the appropriate spread is a decreasing function of h. Prizes $\tilde{W}_1$ and $\tilde{W}_2$ must also satisfy the zero-profit constraint $\tilde{W}_1 + \tilde{W}_2 = V \cdot (\mu_a^* + \mu_b^*)$ independent of h since the spread is always adjusted to induce investments μ_a^* and μ_b^*.

The gain to an a from playing a b with handicap h, rather than another a with no handicap, is the difference in expected prizes:

$$\gamma_a(h) = \bar{P}\tilde{W}_1 + (1 - \bar{P})\tilde{W}_2 - C_a(\mu_a^*) - [(W_1^a + W_2^a)/2 - C_a(\mu_a^*)] \quad (30)$$
$$= \bar{P}\tilde{W}_1 + (1 - \bar{P})\tilde{W}_2 - (W_1^a + W_2^a)/2,$$

where $\gamma_a(h)$ is the gain to a and $\bar{P} = G(\Delta\mu - h)$ is the probability that a wins the mixed match. The corresponding expression for b is

$$\gamma_b(h) = (1 - \bar{P})\tilde{W}_1 + \bar{P}\tilde{W}_2 - (W_1^b + W_2^b)/2. \quad (31)$$

The zero-profit constraints in a-a, a-b, and b-b require that $\gamma_a(h) + \gamma_b(h) = 0$ for all admissible h. The gain of playing mixed matches to a is completely offset by the loss to b and vice versa.

If $C_a(\mu)$ is not greatly different from $C_b(\mu)$, then $\Delta\mu = \mu_a^* - \mu_b^*$ is small and $\bar{P} \doteq \frac{1}{2} + [g(\Delta\mu - h)](\Delta\mu - h)$. This approximation and the zero-profit constraint reduce (30) to

$$\gamma_a(h) \doteq V \cdot \left(\frac{\Delta\mu}{2} - h\right). \quad (32)$$

The expression for $\gamma_b(h)$ is the same, except its sign is reversed, so the gain to a decreases in h, and the gain to b increases in h. Therefore, $h^* = \Delta\mu/2$ is the competitive handicap, since it implies $\gamma_a(h^*) = \gamma_b(h^*) = 0$. If the actual handicap is less than h^*, then γ_a is positive and a's prefer to play in mixed contests rather than with their own type, while b's prefer to play with b's only. The opposite is true if $h > h^*$.

A two-player game is said to be fair when the players are handicapped to equalize the medians. The competitive handicap does not result in a fair game, since $h^* = \Delta\mu/2 < \Delta\mu$. The a's are given a competitive edge in equilibrium, because they contribute more to total output in mixed matches than the b's do. This same result holds if ϵ_a has a different variance than ϵ_b, but it may be sensitive to the assumption of statistical independence and output additivity.

Alternatively, h can be constrained to be zero. In this case, different wage schedules would clear the market. Since $\gamma_a(0) = -\gamma_b(0) \equiv \beta$, paying $\tilde{W}_1 - \beta$ and $\tilde{W}_2 - \beta$ to a's, while paying $\tilde{W}_1 + \beta$, $\tilde{W}_2 + \beta$ to b's, leaves the spread and, therefore, the investments unaltered. It is easy to verify that a's and b's are still indifferent between mixed and pure contests, because expected returns are equal between segregated and integrated contests for each type of player. With no handicaps, the market-clearing prizes available to a's in the mixed contest are lower

than those faced by b's. Still, expected wages are higher for a's than b's in the mixed contest, because their probability of winning is larger. The b's are given a superior schedule in the mixed contest as an equalizing difference for having to compete against superior opponents. This yields the surprising conclusion that reverse discrimination, where the less able are given a head start or rewarded more lucratively if they happen to accomplish the unlikely and win the contest, can be consistent with efficient incentive mechanisms and might be observed in a competitive labor market.

V. Summary and Conclusions

This paper analyzes an alternative to compensation based on the level of individual output. Under certain conditions, a scheme which rewards rank yields an allocation of resources identical to that generated by the efficient piece rate. Compensating workers on the basis of their relative position in the firm can produce the same incentive structure for risk-neutral workers as does the optimal piece rate. It might be less costly, however, to observe relative position than to measure the level of each worker's output directly. This results in paying salaries which resemble prizes: wages which differ from realized marginal products.

When risk aversion is introduced, the prize salary scheme no longer duplicates the allocation of resources induced by the optimal piece rate. Depending on the utility function and on the amount of luck involved, one scheme is preferred to the other. An advantage of a contest is that it eliminates income variation which is caused by factors common to workers of a given firm.

Finally, we allow workers to be heterogeneous. This complication adds an important result: Competitive contests do not automatically sort workers in ways that yield an efficient allocation of resources when information is asymmetric. In particular, low-quality workers attempt to contaminate firms composed of high-quality workers, even if there are no complementarities in production. Contamination results in a general breakdown of the efficient solution if low-quality workers are not prevented from entering. However, when player types are known to all, there exists a competitive handicapping scheme which allows all types to work efficiently within the same firm.

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