International Journal of Industrial Organization 61 (2018) 155–191

Contents lists available at ScienceDirect

International Journal of Industrial Organization

www.elsevier.com/locate/ijio

Information Acquisition, signaling and learning in duopoly

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A R T I C L E   I N F O

Article history:
Received 23 July 2017
Revised 29 July 2018
Accepted 5 August 2018
Available online 18 September 2018

JEL classification:
D4
D8
L1

Keywords:
Information acquisition
Signaling
Information sharing
Product differentiation

A B S T R A C T

We study firms' incentives to acquire private information on cost in a duopoly signaling game. Firms first choose how much to invest in information acquisition and then engage in dynamic price competition. In equilibrium firms acquire too little information from the perspective of industry profit and the perspective of social welfare. We consider two policies that an industry trade association may institute in light of this: (i) the trade association invests directly to acquire private information for each firm, and (ii) firms individually invest in acquiring private information on their costs, and the trade association collects this information and disseminates it after first period prices have been set. Allowing the trade association to acquire information increases firms' profits and may also increase consumer surplus. Information sharing eliminates firms' signaling incentives, and as a result leads to more information acquisition by the firms and higher consumer surplus as well as higher social welfare. However, information sharing increases firms' profits only when the *ex ante* uncertainty about cost is large, and it reduces profits otherwise.

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https://doi.org/10.1016/j.ijindorg.2018.08.003

1. Introduction

Firms' actions often reflect their private information. In dynamic imperfect competition, rival firms frequently extract information relevant for future competition from each other's price or output decisions and use this information as a basis to determine future strategies. An implication of this is that firms may have an incentive to deliberately distort their actions to soften future competition, in recognition of their rivals drawing inferences about their underlying private information.

The canonical models of such signaling incentives postulate that firms are exogenously endowed with some private information on either costs or demand. For instance, Mailath (1989) and Bonatti et al. (2017) assume that firms are endowed with private information on costs, whereas Caminal (1990); Gal-Or (1987); Jin (1994), and Mailath (1993) assume firms are endowed with private information on demand. In practice, however, rather than being exogenously endowed with such private information, firms often must make conscious and costly decisions to acquire such information, and the quality of this information hinges on the amount of resources devoted by firms. For example, during the development of new products, firms often face cost or demand uncertainties and, thus, base their prices on forecasts.[1] Firms who hire managers with rich experience in operation or use the most up-to-date techniques to track and forecast costs or expend resources on demand estimates are more likely to make an accurate estimate of their actual costs or demand.

While the incentive to distort prices due to signaling has been extensively studied and is well understood, it is unclear how signaling affects firms' initial information acquisition decisions. In this paper, we allow firms' private information to be chosen endogenously and focus on their information acquisition incentives in signaling games. Specifically, we address the following questions: How much information do firms acquire when their prices signal information to rivals? How do firms' information acquisition decisions differ from those of an industry-wide trade association or from those of a social planner? How does information sharing affect firms' information acquisition, profits, and consumer welfare?

In order to address these questions, we consider two firms that produce differentiated products and compete in prices in two periods. The firms' idiosyncratic costs are constant across time and are initially unknown to them.[2] Prior to the first period competition, firms simultaneously invest in information acquisition which results in (noisy) private signals about their costs. Thereafter, firms simultaneously choose prices that are public information, but profits remain private information. Going into the second period, firms learn their own costs and glean information about their rivals' costs through their first period prices. In the second period, firms compete in price again and their second period profits are realized.

When a firm invests to improve the accuracy of its initial cost forecast, a first period gain is immediate: More accurate forecasts on costs help firms better attune their

[1] See, e.g., Duane et al. (2010) or Jansen (2008).
[2] In Section 5 we show that many findings also apply to quantity competition and demand uncertainty.

first period prices to actual costs. In contrast, because firms observe their actual costs on the basis of the first period outcomes, more accurate initial cost estimates do not improve cost information in the second period. Nevertheless, increased accuracy of initial cost-estimates does impact the second period market equilibrium: In particular, more accurate first period estimates increase the correlation between firms' second period prices, and this—we show—adversely affects firms' second period profits.

To see why the accuracy of firms' private signals increases the correlation between second period prices, consider the case in which goods are substitutes.[3] The more accurate a firm's initial cost forecast is, the stronger is the positive correlation between its prices across time. Hence, its rival is more confident that the firm will charge a high second period price after observing its high first period price, and since prices are strategic complements, the rival is more likely to also charge a high price in the second period. This increased price correlation is harmful for firms in the second period under cost uncertainty, which echoes the finding in Gal-Or (1986) who studies firms' incentives to share cost information prior to price competition and shows that firms will conceal such information to reduce price correlation. While firms in Gal-Or (1986) are exogenously endowed with private information and compete only in one period, we consider a model with dynamic competition and focus on how signaling affects the amount of private information acquired by firms.

Despite the opposing effects across the two periods, firms benefit from information acquisition, and the degree of information acquisition is uniquely determined by balancing the gain in total profit against the cost of information acquisition. The more independent the two goods are (i.e., weaker substitutes or weaker complements) the more firms invest in information acquisition as the negative second period effects tied to price correlation are diminished. Concerning welfare, we find that from an industry-wide perspective, firms acquire too little information because they fail to internalize the positive externality of their information on their rivals' second period profits. Firms also acquire too little information compared to the socially desirable level of information acquisition. We consider two policies that affect information acquisition and explore their welfare implications.

We first consider a trade association that directly acquires private cost information on behalf of the firms, but does not institute information sharing nor intervene in the subsequent price competition. Since the trade association fully internalizes the positive externalities of the firms' information on their rivals, it acquires more information than the firms when they make the investment independently. Indeed, unless the goods are closely related (either strong substitutes or strong complements), even the trade association's preferred level of investment in information acquisition is too low compared to the social optimum—showing that industry-wide coordinated information acquisition may be pro-competitive.

[3] The case of complements is analogous and follows readily once it is recognized that prices are strategic substitutes, so more accurate information increases the (negative) correlation of second period prices.

Second, we suppose that firms make their own independent investments in obtaining private information, but the trade association collects and disseminates the private signals on costs after firms have set their first period prices. This information sharing arrangement eliminates firms' signaling incentives without affecting other aspects of the interaction. Thus, it allows the identification of the extent to which firms' information acquisition is affected by price signaling. We find that firms acquire more information under information sharing.[4] To understand this result, first note that signaling results in a price distortion in the first period (see, e.g., Mailath, 1989; Caminal, 1990; Gal-Or, 1987; Jin, 1994; Bonatti et al., 2017 and others). Specifically, under cost uncertainty, a firm distorts its first period price above the optimal static level in a (futile) attempt to signal that the firm has a high cost, which—if successful—would soften future competition. This price distortion reduces the value of information accrued to the firms and results in less information acquisition. To see this, note that when a firm acquires more information, its private signal becomes more precise, and therefore the firm assigns a larger weight to its private signal in its first period pricing strategy. This in turn results in a larger variance in the firm's first period price. When the firm's first period price is distorted above the static optimum, a larger price variation reduces the firm's expected first period profit. This is because the firm's first period profit function is concave in its price, so a variation in price will create an asymmetric gain and loss in profit and result in overall diminished expected profits. By contrast, under information sharing, firms choose the static optimal prices. By the Envelope Theorem, a price variation does not result in an expected loss in firm profit. So, firms benefit more from information acquisition in this case.

We find that information sharing increases firm profits when there is a large uncertainty about firms' costs and reduces profits otherwise. On the one hand, information sharing promotes information acquisition which enhances firms' profit. On the other hand, firms lower their first period prices due to the lack of signaling incentives, which reduces their profits. When there is a large uncertainty about costs, information is more valuable and the first effect dominates the second. As a result, information sharing increases firms' profits. Although the impact of information sharing on industry profit is ambiguous, it always enhances social welfare because consumers benefit from more information and the gain in consumer welfare dominates the loss in industry profit.

Our study shows that the trade association is inclined to invest in information acquisition on behalf of the firms, or promote information sharing in markets with large uncertainty on firms' idiosyncratic production costs. Policy makers should allow the trade association to engage in such activities because they increase total welfare. For policy makers who use a consumer welfare standard, information sharing is a more effective policy because it unambiguously increases consumer welfare, whereas the direct invest-

[4] When firms commit to sharing their private information, the incentives to signal are eliminated. See, e.g., Li (1985); Vives (1984); Gal-Or (1985, 1986). Raith (1996) presents a synthesis of these models and Vives (1999) contains a comprehensive overview of the results. See also Jansen (2008) and Ganuza and Jansen (2013) who consider information disclosure after an acquisition decision is made. A critical distinction to our work, however, is that we maintain asymmetric information in the first period; allowing for information exchange only after the first period, but before the second period commences.

ment by the trade association reduces consumer welfare when the two goods are closely related.

There is a related literature on information acquisition in oligopolistic competition, but most papers only involve a single period interaction and consider how information acquisition is affected by the source of uncertainty and the nature of competition.[5] More recent papers, for example Angeletos and Pavan (2007); Colombo et al. (2014); Bernhardt and Taub (2015); Myatt and Wallace (2015, 2018), focus on the efficiency of information acquisition and the usage of information acquired in an environment with both public and private signals. In contrast, our paper examines information acquisition from a different perspective: We study information acquisition in dynamic competition and focus on how signaling incentives affect firms' willingness to invest in information acquisition.

The rest of the paper is organized as follows: Sections 2 and 3 contain the model and the equilibrium. In Section 4 we consider two policies which affect information acquisition and explore their welfare implications. Section 5 considers some extensions and Section 6 concludes. Missing derivations and proofs are collected in the Appendix.

2. The model

Two risk neutral firms, i and j, produce differentiated products and compete in prices in two periods. Firms' constant marginal costs of production, denoted by c_i and c_j, are fixed across time, but their values are initially unknown. Specifically, c_i and c_j are i.i.d. random variables distributed according to $F(\cdot)$, with $E(c_i) = E(c_j) = \mu_c$ and $\text{Var}(c_i) = \text{Var}(c_j) = \sigma_c^2$ denoting the mean and variance, respectively.[6] Define $\tau_c := \frac{1}{\sigma_c^2}$ as the precision of the distribution of a firm's cost.

Prior to the first period price competition, each firm can make a costly investment to acquire a private signal about its own cost. Let $s_i \in S_i$ denote Firm i's signal. Given c_i, signal s_i is drawn from the distribution $G_i(s_i|c_i)$. Define $G_i(s_i) := \int G_i(s_i|c_i)\, dF(c_i)$ as the unconditional distribution of Firm i's signal. The variance of Firm i's signal given c_i is $\text{Var}(s_i|c_i) = \sigma_i^2$, and $\tau_i := \frac{1}{\sigma_i^2}$ denotes the precision of the signal s_i.

Firm i can choose the precision of its signal τ_i at a cost $k(\tau_i)$, where $k(\cdot)$ is a strictly increasing and convex C^2 function on $\mathbb{R}_+$ with $k(0) = 0$, $\lim_{\tau_i \to 0} k'(\tau_i) = 0$ and $\lim_{\tau_i \to \infty} k'(\tau_i) = \infty$. The objective of a firm is to maximize the sum of its expected profits from the two periods net of the costs of information acquisition. For simplicity of exposition, we assume that there is no discounting.

[5] See, e.g., Li et al. (1987); Hwang (1993, 1995); Raju and Roy (2000); Sasaki (2001); Christen (2005). There is some work that considers dynamic models, e.g. Fudenberg and Tirole (1986), Riordan (1985) and Mirman et al. (1994), but there the information itself is generated through the market activity, rather than firms acquiring private information up-front.

[6] All common cost components are assumed to be known and are normalized to zero and we assume that firms face uncertainty on their idiosyncratic costs. The case with demand uncertainty is discussed in Section 5.

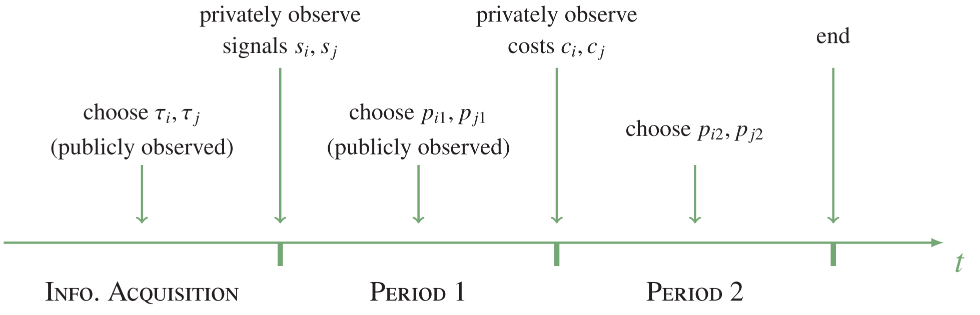

Fig. 1. Timeline.

The representative consumer has a quadratic utility function which takes the form[7]

$$u(q_i, q_j, m) = \eta_0(q_i + q_j) - \frac{1}{2}\left(\eta_1 q_i^2 + 2\eta_2 q_i q_j + \eta_1 q_j^2\right) + m, \tag{1}$$

where m is wealth, q_i, q_j are quantities consumed from the two firms, and η_0, η_1, η_2 are constants with $\eta_0 > 0$, $\eta_1 > |\eta_2| \geq 0$. The two goods are substitutes, independent or complements depending on whether $\eta_2 > 0$, $\eta_2 = 0$, or $\eta_2 < 0$. The two goods are perfect substitutes when $\eta_1 = \eta_2$ and perfect complements when $\eta_1 = -\eta_2$. The coefficient $\frac{\eta_2}{\eta_1} \in (-1, 1)$ is therefore a measure of the degree of product differentiation.

Given the two firms' prices p_{it}, p_{jt} in period $t = 1, 2$, the representative consumer chooses q_{it} and q_{jt} to maximizes her utility, which results in linear demand for each product in period t:[8]

$$q_{it} = a - bp_{it} + ep_{jt}, \tag{2}$$

where $a = \frac{\eta_0}{\eta_1+\eta_2}, b = \frac{\eta_1}{\eta_1^2-\eta_2^2}$ and $e = \frac{\eta_2}{\eta_1^2-\eta_2^2}$.

The timing of the game is illustrated in Fig. 1 and is explained below:

Period 0 information acquisition: Nature draws c_i and c_j independently according to $F(\cdot)$ which is common knowledge. The realizations of c_i and c_j are unknown to both firms. Firms independently choose the precision of the signals, τ_i and τ_j. These choices are publicly observed, but the signals obtained are private information.[9]

Period 1 price competition: Given the firms' private signals about their own costs, s_i and s_j, the firms simultaneously choose publicly observed prices p_{i1} and p_{j1}. After the first period production, markets clear and firms learn their realized costs which remain their private information.

[7] The quadratic utility function is commonly used in the literature to generate linear demand functions; see, for example, Singh and Vives (1984); Caminal (1990); Caminal and Vives (1996).

[8] For exposition ease, we list equations pertaining to Firm i, unless there is a specific reason to list expressions for both firms. The expression for Firm j is symmetrical to that of Firm i.

[9] The assumption of observable choices simplifies the derivation and exposition of the results. In Section 5 we show that our results continue to hold when τ_i, τ_j are unobservable. In particular, we show that a firm's equilibrium choice of precision is unaffected by whether the choice is observed or not.

Period 2 price competition: Firms engage in the second period price competition by choosing p_{i2} and p_{j2} simultaneously.

The following assumption holds for the signal structure.

Assumption 2.1. The information structure satisfies the following conditions:

- *Unbiasedness:* $E(s_i|c_i) = c_i$ and $E(s_j|c_j) = c_j$.
- *Conditional Independence:* Conditional on c_i and c_j, s_i and s_j are independently distributed according to $G_i(s_i|c_i)$ and $G_j(s_j|c_j)$.
- *Affine Posterior Expectation:* $E(c_i|s_i)$ and $E(c_j|s_j)$ are affine in s_i and s_j, respectively.

Several prior-posterior distribution functions give rise to an affine posterior expectation. For example, when $F(c_i)$ is a gamma distribution and $G_i(s_i|c_i)$ is a Poisson distribution with unknown mean c_i; when $F(c_i)$ is a beta distribution and $G_i(s_i|c_i)$ is a binomial distribution; or when $F(c_i)$ and $G_i(s_i|c_i)$ are both normal.[10]

Given Assumption 2.1, Firm i's expected cost conditional on s_i is:[11]

$$E(c_i|s_i) = \overline{\tau}_i s_i + (1 - \overline{\tau}_i)\mu_c, \quad \text{where } \overline{\tau}_i := \frac{\tau_i}{\tau_i + \tau_c}. \tag{3}$$

Thus, a firm's posterior expectation of its cost upon observing signal s_i is a convex combination of the signal and the prior expectation on the cost, μ_c. When Firm i's signal is more precise, the posterior expectation puts a greater weight on the signal compared to the prior mean. As a result, a more precise signal results in a larger dispersion (higher variance) of the conditional expectation $E(c_i|s_i)$.

As learning and conveying information about costs are central to the analysis, in what follows we reserve the use of the expectations operator E to denote expected costs: $E(c|\cdot)$; whereas all other expectations operators are denoted by $\mathbb{E}$, with subscripts denoting the variable with respect to which the expectation is taken.

Strategies and Equilibrium Concept: Firm i's strategy is a triplet $(\tau_i,\ P_{i1}(\cdot),\ P_{i2}(\cdot))$; where $P_{i1}(\cdot)$ and $P_{i2}(\cdot)$ denote pricing functions (whereas p_{i1} and p_{i2} are the chosen prices). Define $\tau := (\tau_i,\ \tau_j)$, and $p_1 := (p_{i1},\ p_{j1})$. For a given p_{j1}, Firm i infers that Firm j's private signal belongs to the set $P_{j1}^{-1}(p_{j1}) := \{s_j | P_{j1}(s_j, \tau) = p_{j1}\}$. If Firm j's first period pricing function is strictly monotone, the set $P_{j1}^{-1}(p_{j1})$ is a singleton and Firm i will perfectly infer s_j.

Given Firm j's strategy $(\tau_j,\ P_{j1}(\cdot),\ P_{j2}(\cdot))$, Firm i's strategy is a best response if

1. $P_{i2}(c_i, p_1, \tau) \in \text{argmax}_{p_{i2}} \mathbb{E}_{c_j}\left[(p_{i2} - c_i)q_{i2}(p_{i2}, P_{j2}(c_j, p_1, \tau)) \mid s_j \in P_{j1}^{-1}(p_{j1}), \tau\right]$, for $p_{j1} \in P_{j1}(S_j,\ \tau)$ and $p_{i1} = P_{i1}(s_i, \tau)$. For a given $(c_i,\ p_1,\ \tau)$, the maximized value of Firm i's second period profit is denoted by $\pi_{i2}(c_i,\ p_1,\ \tau)$.

[10] Affine information structures are common in the literature, see for example Li (1985); Gal-Or (1987, 1988); Chang and Lee (1992).

[11] See, e.g., Ericson (1969).

2. $P_{i1}(s_i,\tau) \in \text{argmax}_{p_{i1}} \mathbb{E}_{s_j}\mathbb{E}_{c_i}[(p_{i1}-c_i)q_{i1}(p_{i1},P_{j1}(s_j,\tau))+\pi_{i2}(c_i,p_{i1},P_{j1}(s_j,\tau),\tau)|s_i,\tau]$. Let $\Pi_i(s_i,\ \tau)$ denote the maximized value of firm *i*'s expected profits from the two periods conditional on s_i and τ.
3. $\tau_i \in \text{argmax}_{\tau_i} \mathbb{E}_{s_i}[\Pi_i(s_i,\tau_i,\tau_j)]$.

The strategy profile $(\sigma_i,\sigma_j) := \{(\tau_i,P_{i1}(\cdot),P_{i2}(\cdot)),(\tau_j,P_{j1}(\cdot),P_{j2}(\cdot))\}$ is a Perfect Bayesian Equilibrium if

1. σ_i is a best response to σ_j and *vice versa*, and
2. for any subsets $\Theta_i \subset S_i$ and $\Theta_j \subset S_j$, $P_{i1}^{-1}(p_{i1}) = \Theta_i$ if $p_{i1} = P_{i1}(\Theta_i)$ and $P_{j1}^{-1}(p_{j1}) = \Theta_j$ if $p_{j1} = P_{j1}(\Theta_j)$.

There are two types of equilibrium configurations. The first type consists of separating configurations in which each firm's first period pricing function is a one-to-one mapping. Hence, $P_{i1}^{-1}(p_{i1})$ and $P_{j1}^{-1}(p_{j1})$ contain a singleton $\forall p_{i1},\ p_{j1}$. The second type consists of (semi-)pooling configurations in which a firm charges the same first period price for different realizations of its signal. These latter configurations entail the firm not always differentiating its action on the basis of the information it obtains. Indeed, in the case of complete pooling, the firm makes no use of information at all; and as information is costly to obtain firms do not engage in any information acquisition.

Our interest is in how firms acquire and make use of information and so we focus on separating configurations in which the incentives to acquire and use information are the strongest. As we then show in Sections 3.4 and 4, firms acquire too little information compared with a social planner or a trade association—a message that is reinforced in pooling or semi-pooling equilibria in which information is of less value to the firms.

3. Equilibrium

Given that we are deriving a separating equilibrium, firms' first period private (noisy) information about their costs is fully revealed by their first period prices. Hence, upon observing Firm *j*'s first period price, Firm i infers that Firm *j*'s private signal is $\hat{s}_j := P_{j1}^{-1}(p_{j1})$, and *vice versa*. Here we use the hat above the signal to denote that this is the inference about the rival's signal, rather than the actual signal. (Consistent beliefs imply that the inference is correct in equilibrium, i.e., $\hat{s}_j = s_j$.)

We proceed with the second period competition, given the firms' conjectures about their rivals' costs; and then move back to characterize firms' first period equilibrium pricing strategies. Finally, we consider firms' information acquisition decisions given the market competition in the continuation equilibrium. Throughout we restrict attention to interior solutions, postulating that the intercept a is sufficiently high to result in positive production.

For expositional convenience, we suppress firms' information acquisition decisions τ when denoting firms' pricing strategies ($P_{it}(\cdot)$, $P_{jt}(\cdot)$) and profit functions ($\pi_{it}(\cdot)$, $\pi_{jt}(\cdot)$). The notation τ is displayed explicitly only in the stage of information acquisition.

3.1. Second period

Each firm learns its own cost after first period production is completed, but the firms still remain uncertain about their rivals' costs. Nevertheless, firms make inferences about their rivals' private signals $\hat{s}_i$ and $\hat{s}_j$ through their first period prices, and use them to update the belief-distribution about their rivals' costs which are given by $F(c_i|\hat{s}_i)$ and $F(c_j|\hat{s}_j)$, respectively. Since $\hat{s}_j$ and $\hat{s}_i$ are based on observable first period prices, they are common knowledge at the outset of the second period.

Firm *i*'s problem in the second period is thus

$$\max_{p_{i2}} \int_{c_j} (a - bp_{i2} + eP_{j2}(c_j))(p_{i2} - c_i) dF(c_j|\hat{s}_j); \tag{4}$$

and Firm *j*'s problem is analogous.

Since firms' second period expected profits are concave in their own prices, first order conditions are both necessary and sufficient, yielding:

$$p_{i2} = \frac{a}{2b} + \frac{c_i}{2} + \frac{e}{2b}\mathbb{E}_{c_j}[P_{j2}(c_j)|\hat{s}_j]. \tag{5}$$

We characterize firms' second period equilibrium prices in the next lemma and use superscript "S" to denote their equilibrium strategies in the signaling game.

Lemma 3.1. *The second period game has a unique Bayesian Nash equilibrium. In the equilibrium, both firms adopt linear pricing strategies. Specifically,*

$$P_{i2}^S(c_i; \hat{s}_i, \hat{s}_j) = \frac{a(2b+e)}{4b^2 - e^2} + \underbrace{\frac{c_i}{2}}_{\text{adaptation effect}} + \underbrace{\frac{beE(c_j|\hat{s}_j)}{4b^2 - e^2} + \frac{e^2 E(c_i|\hat{s}_i)}{2(4b^2 - e^2)}}_{\text{strategic interaction effects}}, \tag{6}$$

$$P_{j2}^S(c_j; \hat{s}_j, \hat{s}_i) = \frac{a(2b+e)}{4b^2 - e^2} + \frac{c_j}{2} + \frac{beE(c_i|\hat{s}_i)}{4b^2 - e^2} + \frac{e^2 E(c_j|\hat{s}_j)}{2(4b^2 - e^2)}. \tag{7}$$

Firms' equilibrium second period prices depend on their own realized costs and each others' expectations on costs. Take Firm *i*'s equilibrium price (6) as an example. The first term reflects how the demand intercept affects Firm *i*'s price. The second term shows by how much Firm i adapts its second period price to its costs c_i. The final two terms capture the strategic interaction between firms' pricing strategies, which depends on both firms' posterior expectations of each other's costs.

When the goods are substitutes (namely $e > 0$), each of the final two terms is positive. Specifically, the third term shows that Firm i raises p_{i2} when it expects an increase in Firm *j*'s cost. This is because Firm i anticipates that Firm j raises p_{j2} due to Firm *j*'s adaptation effect. Since firms' pricing strategies are strategic complements, Firm i increases p_{i2} as well. The last term shows that Firm *i*'s price also increases in its rival's posterior expectation on Firm i's cost. Using the same argument for Firm i, Firm j raises p_{j2} in response to a more optimistic posterior expectation about Firm i's cost. As a consequence, Firm i also increases p_{i2} due to the strategic complementarity effect.

The case of complementary goods differs slightly. Since $e < 0$ for complementary goods, the third term is negative. Note, however, that the final term is positive also for the case of complements: if the rival expects Firm i's cost to be high, it anticipates a higher price from Firm i. Given strategic substitutes, the rival lowers its price accordingly, which—again due to strategic substitutability—leads firm Firm i to increase its price.

In moving forward, denote Firm i's expected second period equilibrium profit as follows:

$$\pi_{i2}^S(c_i, \hat{s}_i, \hat{s}_j) = \left(a - bP_{i2}^S(c_i, \hat{s}_i, \hat{s}_j) + e\mathbb{E}_{c_j}\left[P_{j2}^S(c_j, \hat{s}_i, \hat{s}_j)|\hat{s}_j\right]\right)\left(P_{i2}^S(c_i, \hat{s}_i, \hat{s}_j) - c_i\right), \quad (8)$$

with P_{i2}^S and P_{j2}^S given in Lemma 3.1.

3.2. First period: signaling and belief manipulation

Consider first period price competition. Firm i receives a private signal s_i about its own cost c_i and updates the distribution on c_i to $F(c_i|s_i)$. The information available to Firm i in this stage is s_i, τ_i, τ_j. Firm i expects its rival's first period price p_{j1} to be a function of the rival's private signal s_j. Conditional on signal s_i, Firm i's expected first period profit from charging p_{i1} is given by

$$\begin{aligned}\Pi_{i1}(p_{i1}, P_{j1}|s_i) :&= \int_{s_j}\int_{c_i}(a - bp_{i1} + eP_{j1}(s_j))(p_{i1} - c_i)dF(c_i|s_i)dG_j(s_j)\\ &= \left(a - bp_{i1} + e\mathbb{E}_{s_j}[P_{j1}(s_j)]\right)(p_{i1} - E(c_i|s_i)). \quad (9)\end{aligned}$$

Conditional on s_i, Firm i's expected second period profit from charging p_{i1} is

$$\Pi_{i2}^S(\hat{s}_i|s_i) := \int_{c_i}\int_{\hat{s}_j}\pi_{i2}^S(c_i, \hat{s}_i, \hat{s}_j)dG_j(\hat{s}_j)dF(c_i|s_i), \quad (10)$$

where $\hat{s}_i = P_{i1}^{-1}(p_{i1})$ and $\hat{s}_j = P_{j1}^{-1}(p_{j1})$.

Firm i's problem in the first period is to choose price p_{i1} to maximize the sum of profits from the two periods:

$$\max_{p_{i1}} \Pi_{i1}(p_{i1}, P_{j1}|s_i) + \Pi_{i2}^S(\hat{s}_i|s_i). \quad (11)$$

Using (9) and (10), the first order condition for Firm i (symmetrically for Firm j) is

$$a + bE(c_i|s_i) - 2bp_{i1} + e\mathbb{E}_{s_j}[P_{j1}(s_j)] + \frac{\partial\Pi_{i2}^S(\hat{s}_i|s_i)}{\partial p_{i1}} = 0. \quad (12)$$

The first four terms reflect the first period (static) pricing concerns, whereas the term $\frac{\partial \Pi_{i2}^S(\hat{s}_i|s_i)}{\partial p_{i1}}$ captures Firm i's distortion of the first period price in order to manipulate the rival's belief about the market environment in the second period. Specifically,

$$\begin{aligned}\frac{\partial \Pi_{i2}^S(\hat{s}_i|s_i)}{\partial p_{i1}} &= \int_{c_i}\int_{\hat{s}_j} \frac{\partial \pi_{i2}^S(c_i,\hat{s}_i,\hat{s}_j)}{\partial p_{i1}} dG_j(\hat{s}_j)dF(c_i|s_i) \\ &= \int_{c_i}\int_{\hat{s}_j} \frac{\partial \pi_{i2}^S(c_i,P_{i2}^S,P_{j2}^S)}{\partial P_{j2}^S}\frac{\partial P_{j2}^S}{\partial E(c_i|\hat{s}_i)}\frac{\partial E(c_i|\hat{s}_i)}{\partial p_{i1}} dG_j(\hat{s}_j)dF(c_i|s_i) \\ &= \int_{c_i}\int_{\hat{s}_j} e(P_{i2}^S-c_i)\frac{be\bar{\tau}_i}{4b^2-e^2}\frac{\partial \hat{s}_i}{\partial p_{i1}} dG_j(\hat{s}_j)dF(c_i|s_i), \end{aligned} \tag{13}$$

where the second equation follows from (8) and the Envelope Theorem, and the third equation follows from (7) and (3).

Assuming for the moment that $P_{i1}(s_i)$ and $P_{j1}(s_j)$ are differentiable (which is confirmed in equilibrium), by the Inverse Function Theorem $\frac{\partial \hat{s}_i}{\partial p_{i1}} = \frac{1}{P'_{i1}(s_i)}$. Given $P_{i2}^S - c_i > 0$, $\frac{\partial \Pi_{i2}^S(\hat{s}_i|s_i)}{\partial p_{i1}} > 0$ if $P'_{i1}(s_i) > 0$. So, when firms' first period prices are strictly increasing in their private signals, firms gain from an upward distortion of their first period prices. To see this, note that when a firm raises its first period price, by the Envelope Theorem, it will affect the firm's expected second period profit through its rival's second period price. Specifically, by raising p_{i1} marginally, Firm i shifts up the rival's posterior expectation of Firm i's marginal cost by[12]

$$\frac{\partial E(c_i|\hat{s}_i)}{\partial p_{i1}} = \frac{\partial E(c_i|\hat{s}_i)}{\partial \hat{s}_i}\frac{\partial \hat{s}_i}{\partial p_{i1}} = \frac{\bar{\tau}_i}{P'_{i1}(\hat{s}_i)}. \tag{14}$$

As a consequence, Firm j will raise its second period price by $\frac{be}{4b^2-e^2}\frac{\bar{\tau}_i}{P'_{i1}(s_i)}$ when the goods are substitutes and reduce its price by the same magnitude when they are complements. In both cases, second period competition is relaxed which benefits both firms (refer to (7)). Thus, Firm i has a first-order gain in its second period profit and a second-order loss in its first period profit by an upward distortion of its first period price.

Next, we consider the equilibrium of the entire two period pricing game by considering the first period optimal strategies for given updated beliefs on firms' own costs. The quadratic utility function (i.e., linear demand) together with linear posterior expectations result in a unique fully revealing equilibrium:

Proposition 1. *For a given pair of* $(\tau_i, \tau_j) >> 0$, *there is a unique fully revealing equilibrium in the continuation game starting in the first period. In the equilibrium, firms' first period pricing functions are given by*

$$P_{i1}^S(s_i) = \alpha_1^S E(c_i|s_i) + \beta_1^S, \tag{15}$$

[12] Note from (3), $E(c_i|\hat{s}_i) = \bar{\tau}_i\hat{s}_i + (1-\bar{\tau}_i)\mu_c$.

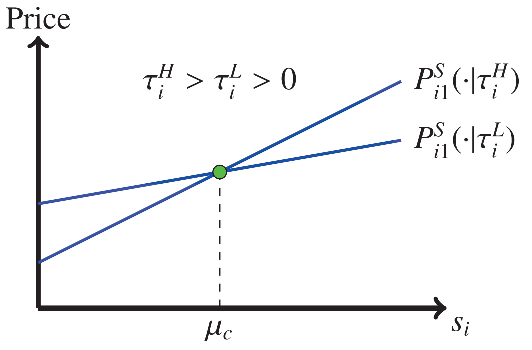

Fig. 2. First period pricing functions for different precision levels.

with $\alpha_1^S = \frac{2b^2-e^2}{4b^2-e^2}$ and $\beta_1^S = \frac{e\mu_c(4b^4-3b^2e^2+e^4)-4ab^2e^2+abe^3+8ab^4+ae^4}{(4b^2-e^2)(2b^2-e^2)(2b-e)}$.

Fig. 2 demonstrates how a firm's equilibrium first period price changes in the precision of its signal.[13] As the signal precision increases, the first period equilibrium price responds more to changes in the realized signal as the pricing function rotates counter-clockwise and becomes steeper. Despite this rotation, the average price remains the same and is $\mathbb{E}_{s_i}(P_{i1}^S(s_i)) = \alpha_1^S \mu_c + \beta_1^S$.

3.3. Information acquisition

Before engaging in price competition in the first period, firms invest in information acquisition and receive unbiased signals about their own costs. When a firm chooses the quality of its signal, it internalizes the impact of its improved private information on its profits in both periods. We analyze how the signal precision affects a firm's first and second period expected profits separately. To begin, we start with the firm's expected first period profit. Define $\Pi_{i1}^S(s_i) := \int_{s_j} \pi_{i1}\big(P_{i1}^S(s_i), P_{j1}^S(s_j)\big) dG_j(s_j)$.

Lemma 3.2. *Firm i's expected first period equilibrium profit is*

$$\mathbb{E}_{s_i}\Pi_{i1}^S(s_i) = \frac{2b^3(2b^2-e^2)}{(4b^2-e^2)^2}\frac{\tau_i}{(\tau_c+\tau_i)\tau_c} + \Phi_1; \tag{16}$$

where Φ_1 *is a collection of terms not involving* τ_i *or* τ_j.

[13] To obtain this figure, one can use (3) to rewrite the equilibrium strategy (15) as $P_{i1}^S(s_i) = \left(\alpha_1^S \frac{\tau_i}{\tau_c+\tau_i}\right)s_i + \left(\alpha_1^S \mu_c \frac{\tau_c}{\tau_c+\tau_i} + \beta_1^S\right)$. Note $\frac{\tau_i}{\tau_c+\tau_i}$ is strictly increasing in τ_i while $\frac{\tau_c}{\tau_c+\tau_i}$ is strictly decreasing in τ_i.

1. *Firm i's expected first period profit increases in the precision of its signal. The marginal gain is*

$$\frac{\partial \mathbb{E}_{s_i} \Pi_{i1}^S(s_i)}{\partial \tau_i} = \frac{2b^3(2b^2 - e^2)}{(4b^2 - e^2)^2} \frac{1}{(\tau_c + \tau_i)^2} > 0. \tag{17}$$

2. *Firm i's expected first period profit is independent of the quality of Firm j's private information. That is,* $\partial \mathbb{E}_{s_i} \Pi_{i1}^S(s_i)/\partial \tau_j = 0$.

In the first period, each firm's price is only based on the conditional expectation of its own cost. Since firms' costs are idiosyncratic, their first period prices are independent. As a result, when a firm increases its investment in information acquisition, the firm can better adjust its first period price to its actual cost, but this does not affect its rival's first period price. The firm therefore has a gain in its expected first period profit.

To see that the firm's expected first period profit is independent of its rival's investment in information acquisition, note that the firm's profit is linear in its rival's price. When Firm j increases the precision of s_j, it increases the variance of its conditional expectation $E(c_j|s_j)$, which in turn increases the variance of Firm j's first period price. However, it leaves unaffected Firm i's expectation of Firm j's price, and as p_{j1} enters Firm i's profit linearly, any gains and losses from variations in p_{j1} offset each other. As a result, Firm i's expected first period profit is independent of Firm j's investment in information acquisition.

The effect of the signal precision on second period expected profits is considerably more nuanced. Note first that upon observing its first period profit, the firm knows its costs with certainty, regardless of the initial choice of precision of the signal it obtained. Thus, there is no incentive to increase the precision of the signal in order to have better information on one's own cost in the second period. However, whereas firms' first period prices are independent, their second period prices are correlated through two channels, and the price correlation increases in firms' investment in information acquisition.

Recall from Lemma 3.1 that each firm's second period price depends on three things: its own realized cost, the rival's posterior expectation of its realized cost, and its posterior expectation of the rival's realized cost. The first channel through which second period prices are correlated is that firms' realized costs are positively correlated with their rivals' conjectures of their realized costs. For example, c_i in (6) is positively correlated with $E(c_i|\hat{s}_i \equiv s_i)$ in (7). And the second source of correlation is that firms' posterior expectations of rivals' costs enter both firms' prices.

While a more precise signal s_i does not affect the firm's knowledge of its own costs c_i, it will increase the correlation between Firm i's realized cost c_i and Firm j's conjecture of Firm i's cost $E(c_i|\hat{s}_i \equiv s_i)$, which in turn, increases the price correlation. The price correlation is reinforced through the second channel. To see this, when Firm i increases the precision of its signal, it increases the variance of $E(c_i|s_i)$ which enters both firms' pricing functions and results in a larger co-movement between firms' second period prices. In short, when firms invest more in information acquisition, their second period prices are more positively correlated for substitutes and more negatively correlated for complements.

Regardless of whether the goods are substitutes or complements, an increase in the correlation of prices reduces firms' second period expected profits. We use substitutes as an example to illustrate the intuition and the case of complements follows the same logic. When a firm has a high cost, due to incomplete pass-through of costs to prices, it will charge a high price but experience a reduction in the profit margin. Since prices are positively correlated, the rival's price is now more likely to also be higher. While this is good news, the good news comes only when the profit-margin is low due to high costs. Now consider a low cost. Due to the increased profit-margin associated with lower costs, the firm optimally lowers its price to increase sales. However, when prices are positively correlated, the rival's price is also more likely to be low and so the firm is unable to take full advantage of its increased profit margin as the rival steals some demand. In sum, when there is co-movement of prices, a rival's price tends to increase a firm's demand when margins are low, and reduces a firm's demand when margins are high.

Thus, when firms' signals become more precise, their second period prices are more correlated and this lowers average second period profits. This finding echoes Gal-Or (1986) who studies information sharing under cost uncertainty. In Gal-Or (1986), firms first decide on the amount of private information to be shared with rivals and then compete in price. She finds that concealing cost information is a dominant strategy in Bertrand competition. In our model, when firms acquire more information, their first period prices become more informative about their costs, which—from the vantage point of the second period—is analogous to sharing more information. Although information acquisition results in a loss in the second period, it yields a gain in the first period. The total benefit depends on the trade off across the two periods and will be analyzed later in this section.

We formalize the above discussion about the impact of information acquisition on firms' second period profits in the following lemma:

Lemma 3.3. *Firm i's expected second period profit is*

$$\mathbb{E}_{s_i}\Pi_{i2}^S(s_i) = -\tfrac{be^2(8b^2-3e^2)}{4(4b^2-e^2)^2}\tfrac{\tau_i}{(\tau_c+\tau_i)\tau_c} + \tfrac{b^3e^2}{(4b^2-e^2)^2}\tfrac{\tau_j}{(\tau_c+\tau_j)\tau_c} + \Phi_2, \tag{18}$$

where the term Φ_2 does not involve τ_i or τ_j.

1. *Firm i's expected second period profit decreases in the precision of its signal. The marginal loss is*

$$\frac{\partial \mathbb{E}_{s_i}\Pi_{i2}^S(s_i)}{\partial \tau_i} = -\frac{be^2(8b^2-3e^2)}{4(4b^2-e^2)^2}\frac{1}{(\tau_i+\tau_c)^2} < 0. \tag{19}$$

2. *Firm i's expected second period profit increases in the precision of Firm j's signal and its marginal gain is*

$$\frac{\partial \mathbb{E}_{s_i} \Pi_{i2}^S(s_i)}{\partial \tau_j} = \frac{b^3 e^2}{(4b^2 - e^2)^2} \frac{1}{(\tau_j + \tau_c)^2} > 0. \tag{20}$$

An implication of Lemma 3.3 is that a firm's investment in information acquisition constitutes a positive externality on its rival's expected second period profit. This is because when a firm's first period price is more informative about its cost, it is easier for the rival to predict the firm's second period price and to best respond accordingly.

Firm *i*'s objective in the information acquisition stage is to maximize the net expected profits from the two periods, namely

$$\max_{\tau_i} \mathbb{E}_{s_i} \left[\Pi_{i1}^S(s_i|\tau_i) + \Pi_{i2}^S(s_i|\tau_i) - k(\tau_i) \right]. \tag{21}$$

Given the convexity of $k(\cdot)$, the objective function is strictly concave in τ_i. We have shown that Firm i gains in the first period but loses in the second period when its signal becomes more accurate. Following (17) and (19), it can be verified that Firm *i*'s total gain from a more precise signal is

$$\frac{\partial \mathbb{E}_{s_i} \Pi_{i1}^S(s_i|\tau_i)}{\partial \tau_i} + \frac{\partial \mathbb{E}_{s_i} \Pi_{i2}^S(s_i|\tau_i)}{\partial \tau_i} = \frac{b(4b^2 - 3e^2)}{4(4b^2 - e^2)} \frac{1}{(\tau_i + \tau_c)^2} > 0. \tag{22}$$

Firm i chooses the precision τ_i to balance its marginal gain in total profit and the marginal cost of information acquisition. The solution is summarized in the following proposition:

Proposition 2. *In the unique fully revealing equilibrium, firms make the same equilibrium information acquisition investment* τ^S*, which is given by*

$$\frac{b(4b^2 - 3e^2)}{4(4b^2 - e^2)} \frac{1}{(\tau_c + \tau^S)^2} = k'(\tau^S). \tag{23}$$

Taking the derivative of the firms' marginal benefits from information acquisition (left hand side of (23)) with respect to the degree of substitution/complementarity between the goods (i.e., e), yields $\frac{-4b^3 e}{(4b^2 - e^2)^2} \frac{1}{(\tau_c + \tau^S)^2}$. This derivative is negative for substitutes and positive for complements, so firms acquire more information when goods are more independent because the negative impact of information acquisition on firms' second period profits diminishes as the correlation between second period prices becomes smaller.

3.4. Welfare

Lemma 3.3 shows that firms' information acquisition has a positive externality on the rivals. Hence, an individual firm's equilibrium information acquisition investment is inefficiently low from the perspective of industry profit. Next, we derive the impact of firms' information acquisition on consumer surplus.

Consumer surplus in period t is denoted by $CS_t = U(q_{it}, q_{jt}, m)$ (see Eq. (1)). Substituting for q_{it}, q_{jt} as defined in (2) into $U(q_{it},\ q_{jt},\ m)$ and using the budget constraint to substitute for m, CS_t is expressed as a function of prices:

$$CS_t(p_{it}, p_{jt}) = \frac{b}{2}(p_{it}^2 + p_{jt}^2) - e(p_{it}p_{jt}) - a(p_{it} + p_{jt}) + \frac{a^2}{b-e}, \tag{24}$$

where a, b and e are defined in (2).

So, the expected consumer surplus in period t can be written as

$$\mathbb{E}[CS_t] = \frac{b}{2}(\mathrm{Var}(P_{it}) + \mathrm{Var}(P_{jt})) - e\mathrm{Cov}(P_{it}, P_{jt}) + \Psi_t, \tag{25}$$

where

$$\Psi_t = \frac{b}{2}\big((\mathbb{E}(P_{it}))^2 + (\mathbb{E}(P_{jt}))^2\big) - e\mathbb{E}(P_{it})\mathbb{E}(P_{jt}) - a(\mathbb{E}(P_{it}) + \mathbb{E}(P_{jt})) + \frac{a^2}{b-e}. \tag{26}$$

Since $\mathbb{E}(P_{it}^S)$ and $\mathbb{E}(P_{jt}^S)$, $t = 1, 2$, do not involve τ_i or τ_j, Ψ_t is a constant not involving τ_i or τ_j.

Lemma 3.4. *The marginal impact of Firm i's information acquisition investment on consumer surplus is*

$$\frac{\partial \sum_{t=1}^{2} \mathbb{E}(CS_t)}{\partial \tau_i} = \frac{b\big(16b^4 - 20b^2e^2 + 3e^4\big)}{4(4b^2 - e^2)^2} \frac{\tau_i}{\tau_c(\tau_i + \tau_c)}, \tag{27}$$

which is positive if $|e| < 0.96b$ *and negative if* $0.96b < |e| < b$.

According to Lemma 3.4, firms' information acquisition investment has a positive externality on consumer welfare as long as the degree of the products' substitutability/complementarity is not too high. Note that firms' information acquisition investment does not directly affect consumer welfare, it indirectly affects consumer welfare through prices. Eq. (25) shows that consumer surplus increases in the variance of prices but decreases in the covariance of firms' prices. When Firm i increases its information acquisition investment, it increases the variance of prices in each period, but also increases the covariance of second period prices because Firm j will glean more precise information about Firm i's cost from its first period price and can better adjust P_{j2} accordingly. When the degree of substitutability/complementarity of the products is not too high, the increase in the price variance dominates the increase in the second period price covariance, yielding a higher consumer surplus. The converse is true when the degree of substitutability/complementarity of the products is high.

A social planner fully internalizes the externality of firms' information acquisition investment and chooses the optimal investment to equate the marginal social benefit and the marginal cost of firms' investments. The socially optimal information acquisition investments are characterized in the following proposition:

Proposition 3. *The social planner's optimal investment in each firm's information acquisition τ^* is uniquely determined by*

$$\frac{b(4b^2 - 3e^2)}{2(4b^2 - e^2)} \frac{1}{(\tau^* + \tau_c)^2} = k'(\tau^*). \tag{28}$$

Firms' equilibrium information acquisition investments are inefficiently low compared to the socially optimal levels.

Even though Firm i's investment in information acquisition reduces consumer welfare when the degree of the products' substitutability/complementarity is very high, the positive externality of Firm i's investment on the rival dominates the negative externality on consumer welfare. Hence, Firm i's equilibrium investment is inefficiently low from the social planner's perspective. Comparing (28) and (23) side-by-side, it is straightforward to see that $\tau^* > \tau^S$.

4. Trade association

The previous section shows that firms' equilibrium information acquisition investment is inefficiently low both from the perspective of industry profit and from the perspective of social welfare. In this section, we investigate the role of a trade association in promoting information acquisition investment. In practice, a trade association does not have the ability to dictate firms' individual information acquisition investment decisions, but a trade association can acquire cost information on behalf of member firms and can then disclose this information to them. Thus, we first study the scenario in which the trade association directly invest in information acquisition.

When it is not feasible for the trade association to directly acquire information, it can still influence firms' information acquisition investment by changing the nature of their price competition. It is illegal for the trade association to directly control firms' prices, but it can influence firms' pricing strategies through an information sharing arrangement. So, the second policy we study is one in which the trade association mandates firms to share their private cost signals prior to the second-period competition. Since the two policies of interest not only affect firm profits but also affect consumer welfare, we study their welfare implications to shed some light on whether a competition authority should be permissive of these activities.

4.1. Acquiring and disclosing information

We first suppose that the trade association directly invests to acquire firms' cost information and discloses to each firm its own cost signal before the first period competition. In keeping with standard anti-trust limitations, the trade association does not interfere in the firms' pricing decisions, leaving the price-competition game unaffected. Specifically, the trade association chooses τ_i and τ_j and reports to each firm its own cost signal in Period 0 of the game. Thereafter, the firms engage in price competition in Periods 1 and 2 as in the signaling game characterized in Sections 3.2 and 3.1.

Since the trade association fully internalizes the positive externality of the investment in each firm's cost information on the other firm, it invests more in information acquisition than the firms when they make the investment independently. The next proposition characterizes the trade association's optimal information acquisition investment.

Proposition 4. *The trade association's optimal investment in information acquisition τ^{TA} is uniquely determined by*

$$\frac{b\big(16b^4-12b^2e^2+3e^4\big)}{4(4b^2-e^2)^2}\frac{1}{(\tau^{TA}+\tau_c)^2}=k'(\tau^{TA}). \tag{29}$$

Clearly, firm profits are higher when the trade association invests in information acquisition. Whether a competitive authority should permit the direct investment by the trade association hinges on its impact on consumer welfare and social welfare, which we address in the next proposition.

Proposition 5. *Allowing the direct investment by the trade association increases consumer welfare when $|e|<0.96b$ and reduces consumer welfare when $0.96b<|e|<b$. Compared with the social planner's optimal information acquisition investment τ^*, the trade association acquires too little information when $|e|<0.96b$ and too much information when $0.96b<|e|<b$.*

The proof follows directly from Lemma 3.4, so it is omitted to avoid repetition. Because the trade association invests more in information acquisition than firms do, the direct investment by the trade association benefits consumers if $|e|<0.96b$ and hurts them, otherwise. When the social planner chooses the optimal τ^*, it fully internalizes the positive externality of each firm's information acquisition on the other firm and its externality on consumers. When $|e|<0.96b$, consumers benefit from more information, but this positive externality on consumers is ignored by the trade association, making the trade association's investment inefficiently low. By contrast, when $0.96b<|e|<b$, more information reduces consumer welfare, and this negative externality is also ignored by the trade association. In this case, the trade association acquires excessive information compared with the socially optimal level. Note that when the trade association acquires excessive information, the direct investment by the trade association may still lead to better outcomes than when the trade associations' activities are prohibited. This is because the gain in firm profits may dominate the loss in consumer welfare, rending a higher social welfare than when the direct investment by the trade association is prohibited.

In the above analysis, the trade association discloses to each firm its own cost signal. One may wonder whether the trade association can further improve industry profit by acquiring information and disclosing each firm's cost signal to both firms prior to the first period competition. In this case, firms will base their first period prices on their own cost signals as well as their rivals' cost signals. Moreover, firms lose signaling incentives because the trade association makes each firm's cost signal public information, and hence firms will choose the static optimal prices. It turns out that industry profit will be lower when the trade association acquires firms' cost information and disseminates the information to both firms.[14] This is because in price competition with cost uncertainty, sharing of cost information increases price correlation which reduces firms' profit (see, for instance, Gal-Or, 1986). Moreover, when the trade association disseminates information to both firms, it will acquire less information, which further reduces industry profit.

4.2. Information sharing

In this subsection, we suppose that the trade association cannot directly acquire information on firms' costs (*ex ante*), but it can require members of the association to share information. In particular, we assume that the trade association collects from its members the signals they obtain on costs and then disseminates this information after the first period prices have been set, but before the second period prices are chosen. This eliminates the incentives for signaling, holding constant other aspects of the interactions between the firms. We consider how information sharing affects firms' incentives to acquire information on costs in Period 0.

We are interested in this policy for two reasons. First, compared to releasing private information before first period prices are set (which also eliminates incentives for price-signaling), the trade association prefers to disseminate information after the prices have been chosen because industry profit is higher when the information sharing occurs later. This is so because information sharing at the beginning of the first period allows firms to base their first period prices on the rivals' cost signals which increases first period price correlation and reduces firms' first period profit.[15]

Second, while the policy eliminates firms' incentives to distort their first period prices due to signaling, the policy preserves the information asymmetry on costs in the first period. So, we can identify to what extent firms' information acquisition is affected by signaling incentives, which informs competition agencies about the welfare consequences of price signaling in settings similar to ours.

For expositional simplicity, in the subsequent analysis, we refer to firms in the signaling model as "signaling" firms and firms in the information sharing regime as "non-signaling" firms, and use the superscript "*NS*" to denote non-signaling firms' equilibrium strategies. We first analyze non-signaling firms' equilibrium information

[14] Detailed analysis is relegated to the Supplementary Appendix B in Jeitschko et al. (2018).
[15] Detailed analysis is relegated to the Supplementary Appendix A in Jeitschko et al. (2018).

acquisition investments, which depend on their equilibrium pricing functions in each period. Note that non-signaling firms' second period pricing functions are the same as signaling firms', because firms correctly infer their rivals' private signals in equilibrium even when they do not share this information.

In the first period, non-signaling firms choose the optimal static prices, which are derived by setting $\frac{\partial \Pi_{i2}^S(\hat{s}_i|s_i)}{\partial P_{i1}} = 0$ in (12) and solving for P_{i1}:

$$P_{i1}^{NS}(s_i) = \alpha_1^{NS} E(c_i|s_i) + \beta_1^{NS}, \tag{30}$$

where $\alpha_1^{NS} = \frac{1}{2}$ and $\beta_1^{NS} = \frac{2a+e\mu_c}{2(2b-e)}$. If we regard firms' pricing strategies as functions of $E(c_i|s_i)$, signaling Firm *i's* optimal first period pricing function (15) is flatter but uniformly higher than the non-signaling Firm *i's* optimal first period pricing function for $\tau_i > 0$.[16]

Given the equilibrium pricing functions characterized in (6) and (30), non-signaling firms choose the level of information acquisition investment to maximize their expected profits from the two periods. We characterize non-signaling firms' information acquisition investments and compare them with that of signaling firms in the following proposition:

Proposition 6. *Non-signaling firms' optimal information acquisition investment is* τ^{NS} *which is determined by*

$$\frac{b(4b^2 - 2e^2)^2}{4(4b^2 - e^2)^2} \frac{1}{(\tau_c + \tau^{NS})^2} = k'(\tau^{NS}). \tag{31}$$

Non-signaling firms acquire more information than signaling firms and the divergence between the amount of information acquired by non-signaling and signaling firms increases in the degree of substitutability or complementarity between the goods.

In Fig. 3, the upward sloping curve indicates firms' marginal costs from information acquisition. The two downward sloping curves $\frac{\partial \mathbb{E}_{s_i} \Pi_i^{NS}(s_i)}{\partial \tau_i}$ and $\frac{\partial \mathbb{E}_{s_i} \Pi_i^{S}(s_i)}{\partial \tau_i}$ are non-signaling and signaling firms' marginal gains from information acquisition, respectively. As is shown in the figure, non-signaling firms acquire more information than signaling firms because they derive a larger marginal gain.

To understand the proposition, note that the reason why non-signaling firms acquire more information than signaling firms is tied entirely to first period profit considerations because non-signaling and signaling firms' second period pricing functions are the same. Consider Firm *i*'s first period equilibrium profit in regime $h \in \{S, NS\}$:

$$\Pi_{i1}^h(s_i) = \big(a - bP_{i1}^h(s_i) + e\mathbb{E}_{s_j}\left[P_{j1}^h(s_j)\right]\big)\big(P_{i1}^h(s_i) - E(c_i|s_i)\big), \tag{32}$$

[16] If $\tau_i = 0$, Firm *i*'s signal is completely uninformative, so Firm *i* does not base its first period price on its realized signals, which leads to a pooling equilibrium. In this case, signaling and non-signaling firms' equilibrium prices coincide.

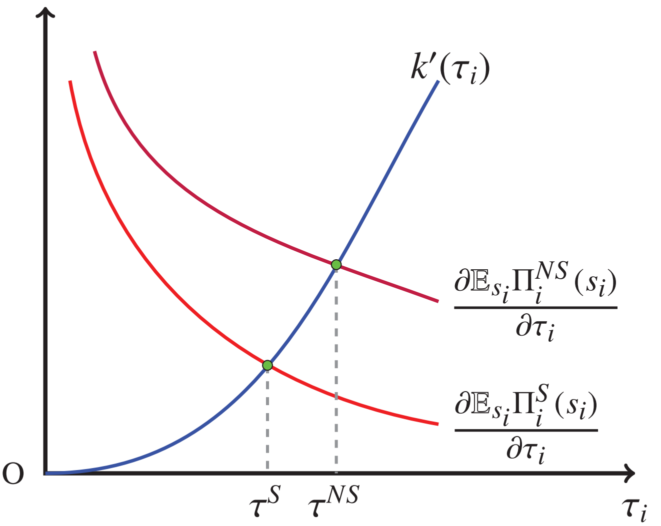

Fig. 3. Comparison of τ^S and τ^{NS}.

where $P_{i1}^S(s_i)$ and $P_{i1}^{NS}(s_j)$ are determined by (15) and (30), respectively. Since Π_{i1}^h depends on the precision of Firm i's signal only through $E(c_i|s_i)$, we can treat Π_{i1}^h as a function of $E(c_i|s_i)$. Hence, to evaluate Firm i's marginal gain from information acquisition in the first period, it suffices to investigate how Π_{i1}^h changes in $E(c_i|s_i)$. Taking the derivative yields:

$$\frac{d\Pi_{i1}^h(E(c_i|s_i))}{dE(c_i|s_i)} = \frac{\partial \Pi_{i1}^h}{\partial E(c_i|s_i)} + \frac{\partial \Pi_{i1}^h}{\partial P_{i1}^h}\frac{\partial P_{i1}^h}{\partial E(c_i|s_i)} \tag{33}$$

$$= -\underbrace{\left(a - bp_{i1}^h(s_i) + e\mathbb{E}_{s_j}\left[P_{j1}^h(s_j)\right]\right)}_{q_{i1}^h} + \frac{\partial \Pi_{i1}^h}{\partial P_{i1}^h}\frac{\partial P_{i1}^h}{\partial E(c_i|s_i)}. \tag{34}$$

The impact of a change in $E(c_i|s_i)$ in Firm i's first period expected profit can be decomposed into a direct effect (the first term in (33)) and an indirect effect through P_{i1}^h (the second term in (33)).

We first analyze the direct effect which equals $-q_{i1}^h$. When Firm i increases the precision of its signal, it also increases the variance of $E(c_i|s_i)$. The question is whether Firm i benefits *ex ante* from a larger variance in $E(c_i|s_i)$. Since P_{i1}^h is strictly increasing in $E(c_i|s_i)$, Firm i reduces the quantity q_{i1}^h when $E(c_i|s_i)$ increases and increases the quantity when $E(c_i|s_i)$ decreases. This implies that the loses to Firm i when $E(c_i|s_i)$ increases are smaller than the gain to the firm when $E(c_i|s_i)$ decreases, and therefore the firm benefits from more accurate private information due to the positive direct impact.

What about the indirect effect through P_{i1}^h? When firms share information, non-signaling firms choose the optimal static prices. By the Envelope Theorem, a change in $E(c_i|s_i)$ does not have an indirect effect on profits and the second term in (33) is 0. In contrast, when firms do not share information, their first period equilibrium prices are distorted upward from the optimal static level due to signaling, and hence the Envelope Theorem does not apply. Specifically, the indirect effect is $\frac{\partial \Pi_{i1}^S}{\partial P_{i1}^S} \frac{\partial P_{i1}^S}{\partial E(c_i|s_i)} = \frac{\partial \Pi_{i1}^S}{\partial P_{i1}^S} \alpha_1^S < 0$. The variation in $E(c_i|s_i)$ induces a loss to non-signaling firms through the indirect effect. To see this, note that Firm i's profit is concave in its own price and P_{i1}^S is greater than the optimal static price. Thus, when P_{i1}^S varies, Firm i's loss from an increase in P_{i1}^S (when expected costs are high) outweighs its gain from a decrease in P_{i1}^S (when costs are low), which results in a loss in profit in expectation. Hence, the distortion in firms' first period prices due to signaling incentives reduces the value of information. When the two goods are closely related, firms have stronger incentives to distort their first period prices to signal high costs. A larger distortion in price makes price variation more costly and hence further reduces the value of information accrued to signaling firms relative to non-signaling firms.

In sum, signaling firms acquire less information than non-signaling firms, because the increased variation in prices caused by more accurate cost estimates harms firms when their prices are distorted above the static optimal prices due to signaling.

Next, we consider how information sharing affects firms' profits.

Proposition 7. *There exists a threshold $\hat{\tau}_c$ such that information sharing increases firms' profits if $\tau_c < \hat{\tau}_c$ and decreases their profits if $\tau_c > \hat{\tau}_c$.*

Information sharing affects firms' profits through two channels. On the one hand, firms lose profit from lower first period prices caused by the elimination of signaling incentives. On the other hand, firms benefit from more precise information due to the increase in information acquisition investment.[17] Hence, whether information sharing increases profit depends on the precision of the prior distribution of cost. When the precision of the prior distribution is low, information is very valuable to firms. As a result, information sharing increases profit because firms' gain from more precise information dominates their loss from lower first period prices. In contrast, if the precision of the prior distribution is high, more information does not have much added value. In this case, information sharing reduces firms' profit because their loss from lower first period prices dominates their gain from more precise information. The next proposition concerns the impact of information sharing on consumer surplus and social surplus.

Proposition 8. *Information sharing increases both consumer surplus and social surplus.*

Information sharing increases consumer surplus for two reasons. First, consumers benefit from lower first period prices. In addition, consumers also benefit from an increase in information acquisition investment. To see this, note that when a firm's signal is more precise, it carries a heavier weight in the firm's pricing strategy in each period, which

[17] Despite the fact that a more precise signal reduces a firm's expected second period profit, the benefit to the firm's first period profit dominates, hence overall, a firm benefits from more precise information.

increases the firm's price variation and in turn increases consumer welfare according to (25). Although more precise information also increases the price covariance in the second period, which reduces consumer welfare as is shown in (25), the variance effect dominates the covariance effect and consumers have a net gain from more precise information.

Combining the results on profit comparison (Proposition 7) and consumer surplus, it is clear that information sharing increases social welfare (net of information acquisition cost) if the precision of the prior distribution is low. When the precision of the prior distribution is high, the gain in consumer surplus dominates the loss in profits, yielding a higher social surplus (net of information acquisition cost). Having postulated that the demand intercept a is sufficiently large throughout the paper, the gain in social welfare from more precise information outweighs the extra cost due to more information acquisition $k(\tau^{NS}) - k(\tau^{S})$.

5. Robustness

We briefly consider which findings are invariant to specific modeling assumptions. In particular, we first consider non-observable levels of investment in information acquisition rather than publicly observable investments. Secondly, we discuss demand uncertainty instead of cost uncertainty. Lastly, we discuss quantity competition, rather than price competition, and show that there are analogous findings concerning the impact of signaling on the value of information in that setting.

5.1. Non-observable investment

For expositional ease of the main analysis, we assumed that firms' investment decisions (namely τ_i, τ_j) are publicly observable. Here we explain why this assumption is without loss of generality and why the equilibrium under observable investments continues to hold with non-observable investments. Lemma 3.2 shows that Firm i's expected equilibrium first period profit does not depend on Firm j's investment in information acquisition. Lemma 3.3 shows that although Firm j's investment increases Firm i's expected equilibrium second period profit, it does not affect the marginal impact of Firm i's investment on its own second period profit. As a result, as shown in Eq. (22), Firm i's total gain from investment in information acquisition is independent of Firm j's investment. So, whether or not Firm i observes the rival's investment does not affect Firm i's equilibrium strategy.

5.2. Demand uncertainty

The main findings are also robust to the source of uncertainty. Specifically, to model demand uncertainty, we assume the two firms' demand intercepts in (2) are subject to i.i.d. random shocks while their marginal costs are common knowledge. A firm can invest in market research to reduce its demand uncertainty and the result of market research is modeled as a conditionally unbiased noisy signal.

Similar to the case with cost uncertainty, a firm distorts its first period price above the optimal static level to “fool” the rival into believing that it faces a strong demand, which will soften second period competition. Again the trade-off is to sacrifice first period profit marginally (second order loss) in exchange for a more favorable second period competition environment (first order gain).

Given that in equilibrium firms infer correctly their rival’s private signal from the first period prices, acquiring more precise information concerning the uncertain demand amounts to sharing more private demand information with rivals. Vives (1984) (Proposition 4) shows that in Bertrand competition with substitutes and demand uncertainty, unilaterally sharing demand information is a dominant strategy for each firm because price correlation increases firm profits in this environment. Therefore, contrary to the case with cost uncertainty (cf. Lemma 3.3), with demand uncertainty, more investment in market research (and hence more precise signals about demand) increases a firm’s expected second period profit. Since firms benefit from more information in both periods, they have an overall gain from information acquisition.

Similar to the environment with cost uncertainty, firms also acquire too little information from the perspective of industry profit and the perspective of social welfare, because they ignore the positive externality of their investment on the rival and the consumer. Hence, allowing the trade association to collect information on behalf of firms could enhance welfare.

Lastly, the result that signaling reduces firms’ value of information continues to hold for the same reason in the case of cost uncertainty. When firms’ private information on demand is more accurate, their first period prices assign a higher weight to their private information, which results in a larger variation in prices. As long as firms’ first period prices are not at the optimal static level due to signaling incentives, their gain in information will be dampened by price variation. Based on this observation, we conclude that information sharing under demand uncertainty will also induce firms to acquire more information.

5.3. Quantity competition

It is well-known that when switching from Bertrand competition to Cournot competition, many qualitative results are reversed. A case in point is how signaling with private information on costs plays out between the two modes of competition. Thus, in our setting firms distort prices above their non-signaling levels to suggest higher costs, whereas in Cournot competition with private cost information, signaling leads to firms increasing their output above non-signaling levels in order to suggest lower costs. An implication of the marked difference in the method of signaling across the two modes (feigning high costs in Bertrand competition vs. feigning low costs in Cournot competition) is that whereas in our framework signaling is anticompetitive, quantity signaling is actually welfare enhancing, because it lowers prices, bringing them closer to the perfect competitive level.

Nevertheless, the result that signaling reduces the value of information holds across both modes of competition. The logic for the case of quantity competition is analogous to that in the price signaling model. When firms invest in information acquisition, their signals about costs become more precise. This is to their benefit in terms of first period profit as it brings their choice variable (price or output) more in line with the true underlying state of the world. However, because in both models of competition signaling incentives result in an upward distortion in firms' first period strategic variables above the non-signaling choices, overshooting is more costly than falling short, and firms refrain from obtaining as much information as they otherwise would in order to reduce the variance of their choices. Thus, signaling incentives—whether in price or quantity competition—reduce the value of information and lead to firms acquiring less precise signals about their costs when compared to non-signaling firms.

6. Conclusion

We study firms' incentives to acquire private information in a duopoly signaling model. Overall, firms benefit from more accurate information. However, firms acquire too little information from the perspective of industry profit and the perspective of social welfare.

From the perspective of the industry, the qualities of firms' information are inefficiently low, which is driven by a positive externality of firms' improved information on their rivals' second period profits. When firms acquire more accurate information, consumers benefit in the first period, but suffer a loss in the second period. Overall, firms' more accurate information has a positive effect on consumers when the degree of substitutability or complementarity between the goods is not too high.

From the perspective of a social planner or a competition authority, the qualities of firms' private information are also inefficiently low. As a result, it may be preferable from a social planer's or competition authority's perspective to allow a trade association to acquire information on the firms' behalf, even when the trade association acquires too much information from a total welfare perspective. Similarly, consumer surplus and total welfare also increase if the trade association collects and disseminates private information from its members after the initial information gathering and competition phases. More generally, the model suggests that under large initial uncertainty the gathering and exchange of information facilitated by a trade a association may be pro-competitive on balance.

Appendix

Proof of Lemma 3.1. The first order conditions (5) imply that each firm's second period pricing function is affine in its own cost. We postulate

$$P_{i2}(c_i; \hat{s}_i, \hat{s}_j) = \alpha_{i2} c_i + \beta_{i2}, \quad \text{symmetrically for } P_{j2}(c_j; \hat{s}_i, \hat{s}_j) \tag{35}$$

Using (35) together with FOCs (5), we solve for the unique set of parameters

$$\alpha_{i2} = \alpha_{j2} = \frac{1}{2}, \quad \beta_{i2} = \beta_{j2} = \frac{a(2b+e)}{4b^2 - e^2} + \frac{beE(c_j|\hat{s_j})}{4b^2 - e^2} + \frac{e^2 E(c_i|\hat{s_i})}{2(4b^2 - e^2)}.$$

□

Proof of Proposition 1. In proving the proposition, we first prove a lemma about a necessary property of any revealing (i.e., separating) equilibrium in our setting concerning the generic form of the differential equation that the first period pricing rule must satisfy. Thereafter we prove that any linear pricing rule is unique; and finally we show that there does not exist a non-linear pricing rule satisfying the necessary condition on the differential equation.

To lay the groundwork, define $x_i := E(c_i|s_i)$. Given (3), there is a one-to-one mapping between s_i and $E(c_i|s_i)$. It is without of loss of generality to regard Firm i's first period pricing rule as a function of x_i. Assume that firms' first period pricing functions are differentiable. It follows that,

$$P'_{i1}(s_i) = P'_{i1}(x_i(s_i))x'_i(s_i) = P'_{i1}(x_i)\bar{\tau}_i. \tag{36}$$

□

Lemma 1. *In any fully revealing equilibrium, Firm i's first period pricing rule $P_{i1}(x_i)$ must satisfy the following differential equation:*

$$P'_{i1}(x_i)(M_3 + x_i - 2P_{i1}(x_i)) = M_2 x_i - M_1, \tag{37}$$

where M_1, M_2, M_3 are constants; and

$$P_{i1}(x_{i0}) = \frac{M_3}{2} + \frac{M_1}{2M_2}, \quad \text{for} \quad x_{i0} = \frac{M_1}{M_2}, \quad \text{and} \tag{38}$$

$$\frac{e^4}{4(4b^2 - e^2)^2} < [P'_{i1}(x_i)]^2. \tag{39}$$

Proof of Lemma 1. We first show that any fully revealing equilibrium must satisfy the differential equation (37). Substitute P_{i2} defined in (6) into (13), Firm i's gain from price distortion in the first period is

$$\begin{aligned}\frac{\partial \Pi_{i2}(\hat{s}_i|s_i)}{\partial p_{i1}} &= \frac{be^2}{(4b^2 - e^2)} \frac{\bar{\tau}_i}{P'_{i1}(s_i)} \int_{c_i} \int_{\hat{s}_j} \left(\frac{a(2b+e)}{4b^2 - e^2} - \frac{c_i}{2} + \frac{beE(c_j|\hat{s}_j)}{4b^2 - e^2} + \frac{e^2 E(c_i|\hat{s}_i)}{2(4b^2 - e^2)} \right) \\ &\quad \times dG_j(\hat{s}_j) dF(c_i|s_i) \\ &= \frac{be^2}{(4b^2 - e^2)} \frac{\bar{\tau}_i}{P'_{i1}(s_i)} \left(\frac{a(2b+e) + be\mu_c}{4b^2 - e^2} + \frac{e^2 - 2b^2}{4b^2 - e^2} E(c_i|s_i) \right),\end{aligned} \tag{40}$$

where the second equation is obtained after imposing consistent beliefs $s_i = \hat{s}_i$ and $s_j = \hat{s}_j$. Using $P'_{i1}(s_i) = P'_{i1}(x_i)\bar{\tau}_i$ and substituting (40) and $E(c_i|s_i) = x_i$ into (12), Firm i's first order condition can be written as follows:

$$a + bx_i - 2bP_{i1}(x_i) + e\mathbb{E}_{x_j}(P_{j1}(x_j)) + \frac{be^2}{4b^2-e^2}\frac{1}{P'_{i1}(x_i)}\left(\frac{a(2b+e)+be\mu_c}{4b^2-e^2} + \frac{e^2-2b^2}{4b^2-e^2}x_i\right) = 0$$

which can be written as $P'_{i1}(x_i)[M_3 + x_i - 2P_{i1}(x_i)] = M_2x_i - M_1$, where[18]

$$M_3 = \frac{a + e\mathbb{E}_{x_j}(P_{j1}(x_j))}{b}, \quad M_2 = \frac{e^2(2b^2-e^2)}{(4b^2-e^2)^2}, \quad M_1 = \frac{e^2(a(2b+e)+be\mu_c)}{(4b^2-e^2)^2}. \quad (41)$$

Next, we show that any fully revealing equilibrium must satisfy the initial condition (38). By strict monotonicity, $P'_{i1}(x_i) \neq 0, \forall x_i$. Hence, for the differential equation to hold at $x_i = x_{i0}$, it is necessary that the expression in the bracket on the left hand side of (37) is zero, which implies $P_{i1}(x_{i0}) = \frac{M_3}{2} + \frac{M_1}{2M_2}$.

Condition (39) ensures that the first order condition is sufficient. To see this, following (12), Firm i's second order condition is $-2b + \frac{\partial^2\Pi_{i2}(\hat{s}_i|s_i)}{\partial p_{i1}^2} < 0$. Note further that

$$\begin{aligned}\frac{\partial^2\Pi_{i2}(\hat{s}_i|s_i)}{\partial p_{i1}^2} &= \frac{be^2}{4b^2-e^2}\frac{\bar{\tau}_i}{P'_{i1}(s_i)}\int_{c_i}\int_{\hat{s}_j}\frac{\partial P_{i2}}{\partial \hat{s}_i}\frac{\partial \hat{s}_i}{\partial p_{i1}}dG_j(\hat{s}_j)dF(c_i|s_i)\\ &= \frac{be^4\bar{\tau}_i}{2(4b^2-e^2)^2}\frac{1}{P'_{i1}(s_i)}\int_{c_i}\int_{\hat{s}_j}\left(\frac{\partial E(c_i|\hat{s}_i)}{\partial \hat{s}_i}\frac{\partial \hat{s}_i}{\partial p_{i1}}\right)dG_j(\hat{s}_j)dF(c_i|s_i)\\ &= \frac{be^4}{2(4b^2-e^2)^2}\left(\frac{\bar{\tau}_i}{P'_{i1}(s_i)}\right)^2 = \frac{be^4}{2(4b^2-e^2)^2}\left(\frac{1}{P'_{i1}(x_i)}\right)^2. \end{aligned} \quad (42)$$

The first equation follows from the first line in (40), the second equation follows from (6), and the third equation follows from (3) as well as the inverse function theorem. Substitute (42) into the second order condition we obtain condition (39). □

Given Lemma 1, the proof of the proposition has two steps. Step 1 shows that there is a unique linear equilibrium. Step 2 shows that there does not exist a nonlinear fully revealing equilibrium.

Step 1. Consider arbitrary linear first period pricing strategies

$$P_{i1}(x_i) = \alpha_{i1}x_i + \beta_{i1}, \quad P_{j1}(x_j) = \alpha_{j1}x_j + \beta_{j1} \quad (43)$$

In a linear equilibrium, Firm i's pricing function $P_{i1}(x_i)$ must satisfy (37):

$$M_2x_i - M_1 = \alpha_{i1}[M_3 - 2(\alpha_{i1}x_i + \beta_{i1}) + x_i]$$

[18] Note that while M_3 is a function of $\mathbb{E}_{x_j}(P_{j1}(x_j))$, since $\mathbb{E}_{x_j}(P_{j1}(x_j))$ does not involve P_{i1} or x_i, it is a constant from Firm i's perspective.

$$\Leftrightarrow \quad M_2 x_i - M_1 = \alpha_{i1}(1-2\alpha_{i1})x_i + \alpha_{i1}(M_3 - 2\beta_{i1})$$
$$\Leftrightarrow \quad M_2 x_i - M_1 = \alpha_{i1}(1-2\alpha_{i1})x_i + \alpha_{i1}\left(\frac{a+e(\alpha_{j1}\mu_c+\beta_{j1})}{b} - 2\beta_{i1}\right). \tag{44}$$

We obtain the last equation by substituting $P_{j1}(x_j)$ (43) into M_3 defined in (41). By symmetry, Firm j's pricing rule $P_{j1}(x_j)$ satisfies $M_2 x_j - M_1 = \alpha_{j1}(1-2\alpha_{j1})x_j + \alpha_{j1}(\frac{a+e(\alpha_{i1}\mu_c+\beta_{i1})}{b} - 2\beta_{j1})$.

The parameters α_{i1}, β_{i1} (symmetrically α_{j1} and β_{j1}) must solve the following system of equations:

$$M_2 = \alpha_{i1}(1-2\alpha_{i1}),\ -M_1 = \alpha_{i1}\left(\frac{a+e(\alpha_{j1}\mu_c+\beta_{j1})}{b} - 2\beta_{i1}\right),$$
$$M_2 = \alpha_{j1}(1-2\alpha_{j1}),\ -M_1 = \alpha_{j1}\left(\frac{a+e(\alpha_{i1}\mu_c+\beta_{i1})}{b} - 2\beta_{j1}\right).$$

where M_1 and M_2 are defined in (41). There are two sets of solutions $\{\alpha_{i1}, \beta_{i1}, \alpha_{j1}, \beta_{j1}\}$ for the above system of equation*. However, the initial condition is satisfied and the first order condition is sufficient only at the solution specified in Proposition 1. To see this, given $0 < |e| < b$, the second order condition (39) is satisfied at $\alpha_{i1} = \alpha_{j1} = \frac{2b^2-e^2}{4b^2-e^2}$, but is violated at the other root $\alpha_{i1} = \alpha_{j1} = \frac{e^2}{2(4b^2-e^2)}$.

Step 2. Totally differentiating the differential equation (37) gives,

$$P''_{i1}(x_i)[M_3 + x_i - 2P_{i1}(x_i)] + P'_{i1}(x_i)[1 - 2P'_{i1}(x_i)] = M_2, \tag{45}$$

Evaluate (45) at the initial value $x_i = x_{i0}$, which implies $M_3 + x_{i0} - 2P_{i1}(x_{i0}) = 0$ and yields

$$2[P'_{i1}(x_{i0})]^2 - P'_{i1}(x_{i0}) + M_2 = 0 \quad \Rightarrow \quad P'_{i1}(x_{i0}) = \frac{1 \pm \sqrt{1-8M_2}}{4}. \tag{46}$$

Totally differentiating (45), yields

$$P'''_{i1}(x_i)[M_3 + x_i - 2P_{i1}(x_i)] + 2P''_{i1}(x_i)[1 - 3P'_{i1}(x_i)] = 0 \tag{47}$$

Again, evaluate (47) at x_{i0}, it follows that $P''_{i1}(x_{i0})[1 - 3P'_{i1}(x_{i0})] = 0$.

Given $0 < |e| < b$, $P'_{i1}(x_{i0})$ in (46) is strictly less than 1/3, and hence (47) implies $P''_{i1}(x_{i0}) = 0$.

Repeat the same process, use the initial condition and $P_{i1}^{(n-1)}(x_{i0}) = 0, \forall n = 3, 4, 5, \ldots$, we obtain

$$P_{i1}^{(n)}(x_{i0})[n - 2(1+n)P'_{i1}(x_{i0})] = 0. \tag{48}$$

Because $P'_{i1}(x_{i0}) < \frac{n}{2(1+n)}$, $P_{i1}^{(n)}(x_{i0}) = 0$. Using Taylor expansion, Firm i's first period price function can be written as

$$P_{i1}(x_i) = \sum_{n=0}^{\infty} \frac{P_{i1}^{(n)}(x_{i0})}{n!}(x_i - x_{i0})^n. \tag{49}$$

Given that $P_{i1}^{(n)}(x_{i0}) = 0, \forall n = 2, 3, 4, \ldots$, $P_{i1}(x_i)$ is linear in x_i. □

Proof of Lemma 3.2. Consider a general linear pricing rule $P_{i1}(s_i) = \alpha_1 E(c_i|s_i) + \beta_1$. Recalling (9), conditional on signal s_i, Firm i's expected first period profit is

$$\begin{aligned}\Pi_{i1}(s_i) &= \big(a - bP_{i1}(s_i) + e\mathbb{E}_{s_j}[P_{j1}(s_j)]\big)(P_{i1}(s_i) - E(c_i|s_i)) \\ &= -b(P_{i1}(s_i))^2 + bP_{i1}(s_i)E(c_i|s_i) + \big(a + e\mathbb{E}_{s_j}[P_{j1}(s_j)]\big)(P_{i1}(s_i) - E(c_i|s_i))\end{aligned} \tag{50}$$

Taking expectations over s_i, Firm i's *ex ante* expected first period profit is

$$\begin{aligned}\mathbb{E}_{s_i}\Pi_{i1}(s_i) = -b\mathbb{E}_{s_i}\left[(P_{i1}(s_i))^2\right] + b\mathbb{E}_{s_i}[P_{i1}(s_i)E(c_i|s_i)] \\ +\big(a + e\mathbb{E}_{s_j}[P_{j1}(s_j)]\big)(\mathbb{E}_{s_i}[P_{i1}(s_i)] - \mu_c).\end{aligned} \tag{51}$$

Using the general linear pricing rule, the second item in (51) can be written as

$$\begin{aligned}b\mathbb{E}_{s_i}[P_{i1}(s_i)E(c_i|s_i)] = b\mathbb{E}_{s_i}\left[P_{i1}(s_i)\left(\frac{P_{i1}(s_i)}{\alpha_1} - \frac{\beta_1}{\alpha_1}\right)\right] = \frac{b}{\alpha_1}\mathbb{E}_{s_i}\left[(P_{i1}(s_i))^2\right] \\ -\frac{b\beta_1}{\alpha_1}\mathbb{E}_{s_i}[P_{i1}(s_i)],\end{aligned} \tag{52}$$

where $\mathbb{E}_{s_i}[P_{i1}(s_i)] = \alpha_1\mu_c + \beta_1$. Substitute $\mathbb{E}_{s_i}[P_{i1}(s_i)E(c_i|s_i)]$ and $\mathbb{E}_{s_i}[P_{i1}(s_i)]$ into (51), we have

$$\begin{aligned}\mathbb{E}_{s_i}\Pi_{i1}(s_i) &= b\left(\frac{1}{\alpha_1} - 1\right)\mathbb{E}_{s_i}\left[(P_{i1}(s_i))^2\right] + \Phi_1(\alpha_1, \beta_1) \\ &= b\left(\frac{1}{\alpha_1} - 1\right)\mathrm{Var}(P_{i1}(s_i)) + \Phi_1(\alpha_1, \beta_1) \\ &= b\alpha_1(1 - \alpha_1)\mathrm{Var}(E(c_i|s_i)) + \Phi_1(\alpha_1, \beta_1) \\ &= b\alpha_1(1 - \alpha_1)\frac{\tau_i}{(\tau_c + \tau_i)\tau_c} + \Phi_1(\alpha_1, \beta_1),\end{aligned} \tag{53}$$

where $\Phi_1(\alpha_1, \beta_1) = (a - (\alpha_1\mu_c + \beta_1)(b - e))((\alpha_1 - 1)\mu_c + \beta_1)$. The last equation in (53) follows from

$$\mathrm{Var}(E(c_i|s_i)) = (\bar{\tau}_i)^2\mathrm{Var}(s_i) = (\bar{\tau}_i)^2(\mathbb{E}(\mathrm{Var}(s_i|c_i)) + \mathrm{Var}(\mathbb{E}(s_i|c_i))) = \frac{\tau_i}{\tau_c(\tau_i + \tau_c)}, \tag{54}$$

where the first equation follows from (3), the second equation follows from the law of total variance, and the last equation is obtained after substituting $\bar{\tau}_i$ defined in (3).

Thus, Firm i's marginal gain from an improvement in the quality of its signal is

$$\frac{\partial \mathbb{E}_{s_i}\Pi_{i1}(s_i)}{\partial \tau_i} = b\alpha_1(1-\alpha_1)\frac{1}{(\tau_c+\tau_i)^2}. \tag{55}$$

After substituting $\alpha_1 = \alpha_1^S$ in Proposition 1 into (55), Firm i's marginal gain in its expected equilibrium first period profit is: $\frac{\partial \mathbb{E}_{s_i}\Pi_{i1}^S(s_i)}{\partial \tau_i} = \frac{2b^3(2b^2-e^2)}{(4b^2-e^2)^2}\frac{1}{(\tau_c+\tau_i)^2}$. So, $\partial \mathbb{E}_{s_i}\Pi_{i1}^S(s_i)/\partial \tau_i > 0$ by assumption $b > |e|$. □

Proof of Lemma 3.3. For notational convenience, express firms' second period equilibrium prices (6) and (7) as

$$P_{i2}^S(c_i, s_i, s_j) = z_0 + \frac{c_i}{2} + z_1 E(c_j|s_j) + z_2 E(c_i|s_i), \text{symmetrically for } P_{j2}^S(c_j, s_j, s_i) \tag{56}$$

where $z_0 = \frac{a(2b+e)}{4b^2-e^2}$, $z_1 = \frac{be}{4b^2-e^2}$, and $z_2 = \frac{e^2}{2(4b^2-e^2)}$.

Using the first order condition, Firm i's expected second period equilibrium profit is

$$\Pi_{i2}^S(s_i) = \mathbb{E}_{s_j}\mathbb{E}_{c_i} b[P_{i2}^S(c_i, s_i, s_j) - c_i]^2 \tag{57}$$

$$\mathbb{E}_{s_i}\Pi_{i2}^S(s_i) = \mathbb{E}_{s_i}\mathbb{E}_{s_j}\mathbb{E}_{c_i} b\left(z_0 + \frac{c_i}{2} + z_1 E(c_j|s_j) + z_2 E(c_i|s_i) - c_i\right)^2 \tag{58}$$

$$\begin{aligned} &= b\mathrm{Cov}\left(\frac{1}{2}c_i, z_2 E(c_i|s_i)\right) + b\mathrm{Var}(z_2 E(c_i|s_i)) + b\mathrm{Cov}(z_2 E(c_i|s_i), -c_i) \\ &\quad + b\mathrm{Var}(z_1 E(c_j|s_j)) + \Phi_2 \end{aligned} \tag{59}$$

$$\begin{aligned} &= \frac{e^2 b}{4(4b^2-e^2)}\mathrm{Var}(E(c_i|s_i)) + \frac{e^4 b}{4(4b^2-e^2)^2}\mathrm{Var}(E(c_i|s_i)) \\ &\quad - \frac{e^2 b}{2(4b^2-e^2)}\mathrm{Var}(E(c_i|s_i)) + \frac{b^3e^2}{(4b^2-e^2)^2}\mathrm{Var}(E(c_j|s_j)) + \Phi_2 \end{aligned} \tag{60}$$

$$= -\frac{be^2(8b^2-3e^2)}{4(4b^2-e^2)^2}\frac{\tau_i}{\tau_c(\tau_c+\tau_i)} + \frac{b^3e^2}{(4b^2-e^2)^2}\frac{\tau_j}{\tau_c(\tau_c+\tau_j)} + \Phi_2 \tag{61}$$

In the derivation, we collect all terms that do not involve signal precision into Φ_2. In obtaining (59), we have used the assumption that c_i, c_j are independent. (60) follows from the fact $\mathbb{E}(c_i, E(c_i|s_i)) = \mathrm{Var}(E(c_i|s_i)) + \text{constant}$, where $\mathrm{Var}(E(c_2|c_1))$ given by (54). Clearly $\partial \mathbb{E}_{s_i}\Pi_{i2}^S(s_i)/\partial \tau_i < 0$, $\partial \mathbb{E}_{s_i}\Pi_{i2}^S(s_i)/\partial \tau_j > 0$. □

Proof of Proposition 2. It can be verified that the left hand side of (23) is strictly decreasing and that $\lim_{\tau_i \to 0} \frac{b(4b^2-e^2)(4b^2-3e^2)}{4(4b^2-e^2)^2} \frac{1}{(\tau_c+\tau_i)^2} = +\infty$ whereas $\lim_{\tau_i \to +\infty} \frac{b(4b^2-e^2)(4b^2-3e^2)}{4(4b^2-e^2)^2} \frac{1}{(\tau_c+\tau_i)^2} = 0$. On the other hand, by assumption $k'(\cdot)$ is strictly increasing and continuous, $k'(0) = 0$ and $\lim_{\tau_i \to +\infty} k'(\tau_i) = +\infty$, hence there's a unique solution to (23). □

Proof of Lemma 3.4. Substituting P_{i2}^S in (56) into (25) and applying $\mathrm{Var}(P_{i2}^S) = \mathrm{Var}(P_{j2}^S)$ and $\mathrm{Var}(E(c_i|s_i)) = \mathrm{Var}(E(c_j|s_i)) = \frac{\tau_i^S}{\tau_c(\tau_i^S+\tau_c)}$ (see (54)). By symmetry, we obtain

$$\begin{aligned}
\mathbb{E}(CS_2) &= b\mathrm{Var}(P_{i2}^S) - e\mathrm{Cov}(P_{i2}^S, P_{j2}^S) + \Psi_2 \\
&= [b(z_1^2 + z_2^2 + z_2) - e(z_1 + 2z_1z_2)]\mathrm{Var}(E(c_i|s_i)) + \Psi_2 \\
&= [b(z_1^2 + z_2^2 + z_2) - e(z_1 + 2z_1z_2)]\frac{\tau_i^S}{\tau_c(\tau_i^S + \tau_c)} + \Psi_2 \qquad (62)
\end{aligned}$$

where the second equation is obtained by substituting

$$\begin{aligned}
\mathrm{Var}(P_{i2}^S) &= z_1^2\mathrm{Var}(E(c_j|s_j)) + z_2^2\mathrm{Var}(E(c_i|s_i)) + z_2\mathrm{Cov}(c_i, E(c_i|s_i)) + \Psi_2 \\
&= \left(z_1^2 + z_2^2 + z_2\right)\mathrm{Var}(E(c_i|s_i)) + \Psi_2 \\
&= \left(z_1^2 + z_2^2 + z_2\right)\frac{\tau_i^S}{\tau_c(\tau_i^S + \tau_c)} + \Psi_2 \qquad (63)
\end{aligned}$$

$$\begin{aligned}
\mathrm{Cov}(P_{i2}^S, P_{j2}^S) &= z_1z_2(\mathrm{Var}(E(c_i|s_i)) + \mathrm{Var}(E(c_j|s_j))) \\
&\quad + \frac{z_1}{2}(\mathrm{Cov}(c_i, E(c_i|s_i)) + \mathrm{Cov}(c_j, E(c_j|s_j))) + \Psi_2 \\
&= \left(\frac{z_1}{2} + z_1z_2\right)(\mathrm{Var}(E(c_i|s_i)) + \mathrm{Var}(E(c_j|s_j))) + \Psi_2 \\
&= (z_1 + 2z_1z_2)\frac{\tau_i^S}{\tau_c(\tau_i^S + \tau_c)} + \Psi_2. \qquad (64)
\end{aligned}$$

For notational convenience, we use Ψ_2 denote any constant term not involving τ_i or τ_j.

Now, consider consumer surplus in the first period. Substitute P_{i1}^S in (15) into (25) and use $\mathrm{Var}(P_{i1}^S) = \mathrm{Var}(P_{j1}^S)$, $\mathrm{Cov}(P_{i1}^S, P_{j1}^S) = 0$, we obtain

$$\begin{aligned}
\mathbb{E}(CS_1) &= b\mathrm{Var}(P_{i1}^S) + \Psi_1(\alpha_1^S, \beta_1^S) \\
&= b\left(\alpha_1^S\right)^2\mathrm{Var}(E(c_i|s_i)) + \Psi_1(\alpha_1^S, \beta_1^S) \\
&= b\left(\alpha_1^S\right)^2\frac{\tau_i^S}{\tau_c(\tau_i^S + \tau_c)} + \Psi_1(\alpha_1^S, \beta_1^S), \qquad (65)
\end{aligned}$$

where

$$\Psi_1(\alpha_1^S, \beta_1^S) = (b-e)(\alpha_1^S\mu_c + \beta_1^S)^2 - 2a(\alpha_1^S\mu_c + \beta_1^S) + \frac{a^2}{b-e}, \qquad (66)$$

by (26) and $\mathbb{E}(P_{i1}^S) = \mathbb{E}(P_{j1}^S) = \alpha_1^S \mu_c + \beta_1^S$. Summing up the consumer surplus from the two periods and take the derivative with respect to τ_i, it follows that

$$\begin{aligned}\frac{\partial \sum_{t=1}^{2} \mathbb{E}(CS_t)}{\partial \tau_i} &= \left(b(z_1^2 + z_2^2 + z_2) - e(z_1 + 2z_1 z_2) + b\big(\alpha_1^S\big)^2\right)\frac{\tau^S}{\tau_c(\tau_c + \tau^S)} \\ &= \frac{b(16b^4 - 20b^2e^2 + 3e^4)}{4(4b^2 - e^2)^2}\frac{\tau^S}{\tau_c(\tau_c + \tau^S)}, \end{aligned} \tag{67}$$

where the second equation is obtained after substituting z_1, z_2 in (56) and α_1^S in Proposition 1.

Next, we show (67) is positive if and only if $|e| < 0.96b$. Let $f(e^2) \equiv 16b^4 - 20b^2e^2 + 3e^4$. The sign of (67) is determined by the sign of $f(e^2)$. Given $|e| < b$, we have

$$f(e^2) > 0 \iff e^2 < \frac{b^2(20 - \sqrt{208})}{6} \iff |e| < 0.96b.$$

□

Proof of Proposition 3. The marginal social benefit of Firm i's investment is obtained by adding up Firm i's marginal benefit in (22), Firm j's marginal benefit in (20), (recall that Firm i's investment only affects its rival's second period profit and does not affect the rival's first period profit) and the representative consumer's marginal benefit in (27). Equating the marginal social benefit with the marginal cost of acquiring information, one has $\frac{b(4b^2-3e^2)}{2(4b^2-e^2)}\frac{1}{(\tau^*+\tau_c)^2} = k'(\tau^*)$. □

Proof of Proposition 4. The marginal gain in industry profit from Firm i's investment is $\frac{b(16b^4-12b^2e^2+3e^4)}{4(4b^2-e^2)^2}\frac{1}{(\tau^{TA}+\tau_c)^2}$, which is obtained by adding up Firm i's marginal gain in (22) and Firm j's marginal gain in (20). The trade association's optimal investment equates the marginal gain in industry profit and the marginal cost of Firm i's investment. The solution τ^{TA} is unique under the assumption that $k'(.)$ is continuous, $k'(.) > 0$, $k'(0) = 0$ and $\lim_{\tau_i \to +\infty} k'(\tau_i) = \infty$. □

Proof of Proposition 6. We first prove that there is a unique solution for non-signaling firms' optimal information acquisition problem. Since non-signaling firms' first period prices are affine in their conditional expectation of costs (refer to (30)), we can use the proof of Lemma 3.2 which shows that when Firm i's first period price is affine in $E(c_i|s_i)$, the firm's marginal gain from an improvement in its private information is (55). Substitute $\alpha_1 = \alpha_1^{NS} = \frac{1}{2}$ into (55), the non-signaling firm's first period gain from a marginal improvement in its private information is $\frac{\partial \mathbb{E}_{s_i}\Pi_{i1}^{NS}(s_i)}{\partial \tau_i} = \frac{b}{4}\frac{1}{(\tau_c+\tau_i)^2}$. Because the expected equilibrium second period profits are the same for signaling and non-signaling firms, it follows that $\partial \mathbb{E}_{s_i}\Pi_{i2}^{NS}(s_i)/\partial\tau_i = \partial \mathbb{E}_{s_i}\Pi_{i2}^{S}(s_i)/\partial\tau_i$, which is given by (19). Thus, it follows that $\partial \mathbb{E}_{s_i}\Pi_{i}^{NS}(s_i)/\partial\tau_i = \partial \mathbb{E}_{s_i}\Pi_{i1}^{NS}(s_i)/\partial\tau_i + \partial \mathbb{E}_{s_i}\Pi_{i2}^{NS}(s_i)/\partial\tau_i = \frac{b(4b^2-2e^2)^2}{4(4b^2-e^2)^2}\frac{1}{(\tau_c+\tau_i)^2} > 0$.

Firm *i*'s optimal signal precision is the solution to

$$\frac{\partial \mathbb{E}_{s_i} \Pi_i^{NS}(s_i)}{\partial \tau_i} = \frac{b(4b^2 - 2e^2)^2}{4(4b^2 - e^2)^2} \frac{1}{(\tau_c + \tau_i)^2} = k'(\tau_i). \tag{68}$$

Note that $\partial \mathbb{E}_{s_i} \Pi_i^{NS}(s_i)/\partial \tau_i$ is decreasing in τ_i, positive at $\tau_i = 0$ and approaching 0 as $\tau_i \to +\infty$. Given the assumptions $k''(\cdot) > 0$, $k'(0) = 0$ and $\lim_{\tau_i \to +\infty} k'(\tau_i) = \infty$, there exists a unique solution $\tau^{NS} \in (0, \infty)$ to Eq. (68).

To compare information acquisition decisions of signaling and non-signaling firms, it suffices to compare the marginal gain in firms' first period profits from an improvement in the quality of their private information. Using (68) and (22), we obtain

$$\frac{\partial \mathbb{E}_{s_i} \Pi_{i1}^{NS}(s_i)}{\partial \tau_i} - \frac{\partial \mathbb{E}_{s_i} \Pi_{i1}^{S}(s_i)}{\partial \tau_i} = \frac{be^4}{4(4b^2 - e^2)} \frac{1}{(\tau_i + \tau_c)^2} > 0, \tag{69}$$

which implies $\tau^S < \tau^{NS}$. Since (69) increases in $|e|$, the degree of substitution between the two goods, the difference in signal precisions $\tau^{NS} - \tau^S$ increases in $|e|$. □

Proof of Proposition 7. For a fixed pair of (τ_i, τ_j), firms' expected second period profits are the same, irrespective of whether they share the cost information. So, signaling and non-signaling firms' second period profits have the same functional form but with different precision τ^S and τ^{NS}. Using (18) and applying $\tau_i^S = \tau_j^S = \tau^S$ by symmetry, signaling Firm *i*'s expected second period profit can be written as

$$\mathbb{E}_{s_i} \Pi_{i2}^{S}(s_i) = -\frac{be^2(4b^2 - 3e^2)}{4(4b^2 - e^2)^2} \frac{\tau^S}{(\tau_c + \tau^S)\tau_c} + \Phi_2. \tag{70}$$

Similarly, Firm *i's* expected second period profit under information sharing is

$$\mathbb{E}_{s_i} \Pi_{i2}^{NS}(s_i) = -\frac{be^2(4b^2 - 3e^2)}{4(4b^2 - e^2)^2} \frac{\tau^{NS}}{(\tau_c + \tau^{NS})\tau_c} + \Phi_2. \tag{71}$$

Now, we analyze the first period. Consider a general affine first period pricing function $P_{i1} = \alpha_1 E(c_i|s_i) + \beta_1$. Using (53), signaling Firm *i's* expected first period profit is

$$\mathbb{E}_{s_i} \Pi_{i1}^{S}(s_i) = b\alpha_1^S(1 - \alpha_1^S) \frac{\tau_i^S}{(\tau_c + \tau_i^S)\tau_c} + \Phi_1(\alpha_1^S, \beta_1^S), \tag{72}$$

where α_1^S and β_1^S are defined in (15). Non-signaling Firm *i's* expected first period profit is

$$\mathbb{E}_{s_i} \Pi_{i1}^{NS}(s_i) = b\alpha_1^{NS}(1 - \alpha_1^{NS}) \frac{\tau_i^{NS}}{(\tau_c + \tau_i^{NS})\tau_c} + \Phi_1(\alpha_1^{NS}, \beta_1^{NS}), \tag{73}$$

where α_1^{NS} and β_1^{NS} are defined in (30).

Summing up Firm i's expected profit from the two periods in regime $h, h \in \{S, NS\}$, we obtain

$$\mathbb{E}\Pi_i^h(\tau^h; \tau_c) = A^h \frac{\tau^h}{(\tau_c + \tau^h)\tau_c} + \Phi_2 + \Phi_1(\alpha_1^h, \beta_1^h), \tag{74}$$

where $A^h = -\frac{be^2(4b^2-3e^2)}{4(4b^2-e^2)^2} + b\alpha_1^h(1-\alpha_1^h)$. Define the difference between signaling and non-signaling Firm i's net profit by $\Delta := [\mathbb{E}\Pi_i^S(\tau^S; \tau_c) - k(\tau^S)] - [\mathbb{E}\Pi_i^{NS}(\tau^{NS}; \tau_c) - k(\tau^{NS})]$. We first establish that Δ is monotone in τ_c. By the Envelope Theorem,

$$\begin{aligned} \frac{\partial \Delta}{\partial \tau_c} &= \frac{\partial \mathbb{E}\Pi_i^S(\tau^S; \tau_c)}{\partial \tau^c} - \frac{\partial \mathbb{E}\Pi_i^{NS}(\tau^{NS}; \tau_c)}{\partial \tau^c} \\ &= -A^S \frac{\tau^S(2\tau_c + \tau^S)}{[\tau_c(\tau_c + \tau^S)]^2} + A^{NS} \frac{\tau^{NS}(2\tau_c + \tau^{NS})}{[\tau_c(\tau_c + \tau^{NS})]^2}, \end{aligned} \tag{75}$$

where the second equation is obtained by taking the derivative of (74) with respect to τ_c. Since $\alpha^{NS} = \frac{1}{2} > \alpha^S = \frac{2b^2-e^2}{4^2-e^2} > 0$, we have $A^{NS} > A^S > 0$. Moreover, $\frac{\tau^{NS}(2\tau_c+\tau^{NS})}{[\tau_c(\tau_c+\tau^{NS})]^2} > \frac{\tau^S(2\tau_c+\tau^S)}{[\tau_c(\tau_c+\tau^S)]^2} > 0$ because $\frac{\tau^h(2\tau_c+\tau^h)}{[\tau_c(\tau_c+\tau^h)]^2}$ is a strictly increasing function of τ^h and $\tau^{NS} > \tau^S$. Hence, $\frac{\partial \Delta}{\partial \tau_c} > 0$.

Lastly, we prove the existence of the unique cutoff $\hat{\tau}_c$. Substituting (74) into Δ, it follows that

$$\begin{aligned} \Delta &= A^S \frac{\tau^S}{(\tau_c + \tau^S)\tau_c} + \Phi_1(\alpha_1^S, \beta_1^S) - A^{NS} \frac{\tau^{NS}}{(\tau_c + \tau^{NS})\tau_c} - \Phi_1(\alpha_1^{NS}, \beta_1^{NS}) \\ &\quad + k(\tau^{NS}) - k(\tau^S) \\ &= -A^S \frac{\tau^{NS} - \tau^S}{(\tau_c + \tau^{NS})(\tau_c + \tau^S)} - \left(\frac{1}{4} - \alpha_1^S(1-\alpha_1^S)\right) \frac{b\tau^{NS}}{\tau_c(\tau_c + \tau^{NS})} \\ &\quad + \Phi_1(\alpha_1^S, \beta_1^S) - \Phi_1(\alpha_1^{NS}, \beta_1^{NS}) + k(\tau^{NS}) - k(\tau^S). \end{aligned}$$

It can be verified that $\frac{1}{4} > \alpha_1^S(1-\alpha_1^S)$ and $\Phi_1(\alpha_1^S, \beta_1^S) > \Phi_1(\alpha_1^{NS}, \beta_1^{NS})$. Moreover, $k(\tau^{NS}) > k(\tau^S)$ because $\tau^{NS} > \tau^S$. So, $\lim_{\tau_c \to 0} \Delta < 0$ and $\lim_{\tau_c \to \infty} \Delta > 0$. Then, the continuity and monotonicity of Δ imply the existence of a unique solution to $\Delta = 0$. Let this solution be $\hat{\tau}_c$. Clearly for $\tau_c > \hat{\tau}_c$, $\Delta > 0$, otherwise $\Delta < 0$. □

Proof of Proposition 8. Let CS_t^{NS} and SW_t^{NS} denote consumer surplus and social surplus, respectively, in period t under information sharing. We first consider the impact of the information sharing on consumer surplus. Under information sharing, firms' second period pricing functions are the same as in the main model except that the equilibrium information acquisition investment is τ_i^{NS} instead of τ_i^S. Using (62), we obtain

$$\mathbb{E}(CS_2^{NS}) - \mathbb{E}(CS_2^S) = (b(z_1^2 + z_2^2 + z_2) - e(z_1 + 2z_1z_2))\left(\frac{\tau_i^{NS}}{\tau_c(\tau_c + \tau_i^{NS})} - \frac{\tau_i^S}{\tau_c(\tau_c + \tau_i^S)}\right)$$

$$= -\frac{be^2(4b^2+e^2)}{4(4b^2-e^2)^2}\frac{\tau_i^{NS}-\tau_i^S}{(\tau_c+\tau_i^{NS})(\tau_c+\tau_i^S)}, \tag{77}$$

where the second equation is obtained by substituting z_1 and z_2 defined in (56).

First period consumer surplus in the signaling game is derived in (65). Similar to the signaling Firm i, non-signaling Firm $i's$ equilibrium first period pricing function is $P_{i1}^{NS}(s_i) = \alpha_1^{NS} E(c_i|s_i) + \beta_1^{NS}$. (See (30)). Using (65), we can express the first period consumer surplus under information sharing as

$$\mathbb{E}(CS_1^{NS}) = b\big(\alpha_1^{NS}\big)^2 \frac{\tau_i^{NS}}{\tau_c(\tau_i^{NS}+\tau_c)} + \Psi_1(\alpha_1^{NS},\beta_1^{NS}). \tag{78}$$

Substituting $\alpha_1^S, \beta_1^S, \Psi_1(\alpha_1^S,\beta_1^S)$ and $\Psi_1(\alpha_1^{NS},\beta_1^{NS})$, we obtain

$$\begin{aligned}\mathbb{E}(CS_1^{NS}) - \mathbb{E}(CS_1^S) &= b\left(\big(\alpha_1^{NS}\big)^2\frac{\tau_i^{NS}}{\tau_c(\tau_i^{NS}+\tau_c)} - \big(\alpha_1^S\big)^2\frac{\tau_i^S}{\tau_c(\tau_i^S+\tau_c)}\right) + \Psi_1(\alpha_1^{NS},\beta_1^{NS}) \\ &\quad -\Psi_1(\alpha_1^S,\beta_1^S) \\ &= \frac{b}{4}\frac{\tau^{NS}-\tau^S}{(\tau^{NS}+\tau_c)(\tau^S+\tau_c)} + \frac{be^2(8b^2-3e^2)}{4(4b^2-e^2)^2}\frac{\tau^S}{\tau_c(\tau_c+\tau^S)} \\ &\quad + \frac{b^2e^2(a-(b-e)\mu_c)^2(8b^3-4b^2e-5be^2+3e^3)}{(2b-e)^4(2b^2-e^2)^2}.\end{aligned} \tag{79}$$

Summing up (77) and (79), it follows that:

$$\begin{aligned}\sum_{t=1}^2 \mathbb{E}(CS_t^{NS}) - \sum_{t=1}^2 \mathbb{E}(CS_t^S) &= \frac{b^3(4b^2-3e^2)}{(4b^2-e^2)^2}\frac{\tau^{NS}-\tau^S}{(\tau_c+\tau^{NS})(\tau_c+\tau^S)} \\ &\quad + \frac{be^2(8b^2-3e^2)}{4(4b^2-e^2)^2}\frac{\tau^S}{\tau_c(\tau_c+\tau^S)} \\ &\quad + \frac{b^2e^2(a-(b-e)\mu_c)^2(8b^3-4b^2e-5be^2+3e^3)}{(2b-e)^4(2b^2-e^2)^2} \\ &> 0.\end{aligned} \tag{80}$$

Next, consider the impact of information sharing on social welfare. The difference between signaling and non-signaling Firm $i's$ net profit is derived in (76). Adding up (76) and (80), the difference in expected social welfare is

$$\begin{aligned}&\sum_{t=1}^2 \mathbb{E}(SW_t^{NS}) - \sum_{t=1}^2 \mathbb{E}(SW_t^S) \\ &= 2\big[\big(\mathbb{E}\Pi_i^{NS}(\tau^{NS};\tau_c) - k(\tau^{NS})\big) - \big(\mathbb{E}\Pi_i^{NS}(\tau^{NS};\tau_c) - k(\tau^{NS})\big)\big] + \sum_{t=1}^2 \mathbb{E}(CS_t^{NS}) \\ &\quad - \sum_{t=1}^2 \mathbb{E}(CS_t^S)\end{aligned}$$

$$
\begin{aligned}
&= \frac{b(12b^4 - 9b^3e^2 + 2e^4)}{(4b^2 - e^2)^2} \frac{\tau^{NS} - \tau^S}{(\tau_c + \tau^{NS})(\tau_c + \tau^S)} + \frac{be^2(8b^2 - e^2)}{4(4b^2 - e^2)^2} \frac{\tau^S}{\tau_c(\tau_c + \tau^S)} \\
&\quad + \frac{be^2(a - (b - e)\mu_c)^2(8b^4 - 12b^3e + b^2e^2 + 5be^3 - 2e^4)}{(2b - e)^4(2b^2 - e^2)^2} \\
&\quad + 2[k(\tau^S) - k(\tau^{NS})]. \qquad (81)
\end{aligned}
$$

One can verify all terms in (81), except for the last item $2[k(\tau^S) - k(\tau^{NS})]$, are positive. This implies that without considering the difference in information acquisition costs, the expected social welfare is higher under non-signaling regime. Since we assume in our paper the demand intercept a is sufficiently large, the term $\frac{be^2(a-(b-e)\mu_c)^2(8b^4-12b^3e+b^2e^2+5be^3-2e^4)}{(2b-e)^4(2b^2-e^2)^2}$ is large, so it dominates the extra cost of information acquisition $2[k(\tau^{NS}) - k(\tau^S)]$. □

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