

Optimal Incentive Contracts with Imperfect Information*

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1. INTRODUCTION

It is well known that in situations involving uncertainty, the existence of a complete set of contingent claims is sufficient to assure a Pareto-efficient allocation of resources. In such situations, however, absence of information or its asymmetrical distribution among the agents can lead to the absence of a complete set of contingent claims. In this case, efficiency may require investing resources in acquiring information and adopting a more complicated exchange structure. A particular case of interest is one in which one economic agent can observe the joint effects of an unknown state of nature and of decisions by another agent, but cannot observe either the state or the decision separately. When the decision involves costs or benefits to the decision maker, an incentive problem, sometimes referred to as moral hazard, arises. This situation occurs for example when decision-making authority is delegated from one agent to another. Specific examples include such relationships as those between owners and managers of a firm, governments and contractors, insurers and insured, etc. The purpose of this paper is to develop a theory of contracts in situations characterized by a divergence of incentives between the two parties and asymmetric information (i.e., moral hazard) with special emphasis on how the possibilities for acquiring information affect the structure of the contract. In particular, we seek to explain the widespread use in areas such as employment and insurance of contracts in which the result of an imperfect (noisy) monitoring process is used to determine the schedule according to which one agent is compensated by another.

The theory of optimal contracts under conditions of uncertainty has recently received considerable attention. Arrow [2] and Wilson [16] were concerned with the optimal sharing of purely exogenous risk. Wilson [17], Leland [9], and Ross [10, 11] considered situations in which risk could be

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affected by the actions of the agents. They analyze contracts which induce similar attitudes toward risk on the part of the agents thus allowing the possibility that decentralized decision making will be optimal. Conditions under which such arrangements are indeed Pareto-optimal are also investigated. In their models incentive problems arise purely as a consequence of diverse attitudes toward risk among the agents. Spence and Zeckhauser [13], in the context of insurance contracts, introduced the problem of a divergence in incentives due to the action of an agent together with differential information among agents. More recently, Stiglitz [14] analyzed incentive contracts between employers and employees. With regard to differential information, both Spence and Zeckhauser and Stiglitz assume one of two extreme cases: Either the agent's action is known by everyone with certainty (in which case there is no differential information) or no information about the agent's action is available to anyone except the agent himself. A somewhat intermediate case was analyzed by Townsend [15] in which the exact information possessed by one agent can be made available to the other at some cost. Most recently, Shavell [12] and Holmstrom [7] have analyzed models of imperfect monitoring. Holmstrom extends some of our results.

Two important aspects of agency relationships are not fully explored in the literature on the theory of contracts. First, most agency relationships must deal with incentive (moral hazard) problems which arise because the agent would prefer to work less, other things equal, while the principal is indifferent to the level of the agent's effort, other things (i.e., his share of the payoff) equal. This type of incentive problem is somewhat different from the one considered by Wilson [17], Leland [9], and Ross [10] in which a divergence in incentives results only from different attitudes toward risk. Second, in most instances, an agent may acquire information about other agents' actions. This possibility was discussed in interesting papers by Alchian and Demsetz [1] and Jensen and Meckling [8]. The quality of the information obtained through monitoring (or supervising) depends on the resources committed to this activity as well as on the available monitoring technology. Furthermore, as Stiglitz [14, p. 572] points out, "...the amount (or quality) of supervision will affect both the optimal incentive scheme which will be used and the level of expected utility which the individual will attain." Consequently, the optimal incentive contract will depend on the available monitoring technology. In this paper, we explore these aspects of the agency problem.

Our analysis is based on a model in which there are two individuals; one, denoted the agent, takes an action which together with the realization of an exogenous random variable results in the payoff to be divided between the agent and the other individual, denoted the principal. Incentive problems arise because the agent has a disutility for the action while the principal does not. We distinguish two versions of this model. In the first, the agent is assumed to take his action without any information regarding the realization

of the exogenous random state. In the second version, the agent is assumed to know the value of the random state before taking his action. In both versions, the object of the analysis is to discover the form of the Pareto-optimal contract, i.e., how the optimal sharing arrangement for the payoff depends on the observed variables. In particular, our analysis deals with the following issues: When would we expect to observe performance-contingent contracts, and what would be the form of such contracts; when performance is not observable, under what conditions would we expect contracts to depend on imperfect estimates of performance, what types of estimators would be used, and how would they be incorporated into the contract.

Our results are summarized as follows:

(i) There are *no* gains to be derived from monitoring the agent's action when the agent is risk neutral. There are also no gains when there is no uncertainty *ex post* about the relationship between the agent's action and the payoff. In these cases, the optimal risk sharing arrangement resolves the moral hazard problem.

(ii) We characterize the form of the Pareto-optimal contract under various assumptions on the observability of the action and the random state.

(iii) There *are potential* gains to monitoring in all cases other than those described in (i).

With regard to the first version of the model:

(iv) It has been shown by Shavell [12] (and also by Holmstrom [7]) that whenever there are potential gains to monitoring, these gains can be realized by use of a contract which incorporates imperfect monitoring. We characterize the form of the optimal monitoring contract. In particular, we prove that the optimal monitoring contract is of the following form: If the agent's action is judged acceptable on the basis of the monitoring outcome, the agent is paid according to a prespecified schedule. Otherwise, he receives a less preferred, fixed payment (e.g., he is dismissed). This result explains the ubiquitous use of such contracts.

(v) As a basis for further testing the model, we derive explicitly, for a special case, the optimal monitoring contract. The dependence on the precision of the monitor of the agent's compensation, the minimum acceptable action, the equilibrium action, the equilibrium probability of dismissal, and the benefits from monitoring is explicitly derived.

We believe that the models described herein capture several of the important features of many "agency" relationships. In particular, monitoring plays an important role in most such relationships. One example, to which we shall return from time to time, is the employer-employee relationship. Here the

payoff depends on the employee's effort which can be monitored by the employer. Specific versions of the employer–employee situation include patient–doctor, client–lawyer, sales manager–salesman, and owner–manager arrangements, as well as the more traditional forms of supervisor–worker relationships. A second set of examples would include arrangements between insurers and insureds such as automobile or health insurance and product warrantees. A third group of examples involves owners and users of physical (as opposed to human) capital such as landlord–tenant and other lessor–lessee relationships. In [4], we apply our results to several examples.

The paper proceeds as follows. In Section 2, we present our formal model; Section 3 contains results on the structure of optimal contracts and desirability of monitoring. Properties of monitoring technologies which assure the desirability of monitoring and the form of the corresponding contract are derived in Section 4. Section 5 contains concluding remarks.

2. THE MODELS

In this section we describe the two models with which we may analyze the questions set out in the previous section. We consider only two individuals, “the principal” and “the agent” (hereafter referred to as P and A , respectively). The agent takes an action which, together with the realization of some exogenous event, determines a payoff. Other than possibly supervising the agent, the principal is assumed not to take any action which would affect this payoff. The contract which specifies how the payoff is shared is the object of our analysis. This contract must resolve the conflict of interest which arises from the fact that the agent derives disutility from his action while the principal does not.

Somewhat more precisely, the agent is assumed to choose some action, a , which is an element of a compact subset A of the nonnegative reals (whether the symbol A refers to the agent or his action choice set will be clear from the context). The action results in a payoff, x , where

$$x = X(a, \theta), \quad X_1 > 0 \quad \text{for all } a, \theta, \quad (1)$$

and θ is an exogenous random state.

Two important cases may be distinguished regarding the information available to the agent when he chooses his action. First, in Model 1, we assume that he does *not* know the value of θ when he chooses his effort. Second, in Model 2, we assume that the agent *does* observe the value of θ and chooses his effort contingent on this observation. In both cases, the contract is negotiated before the realization of the state and both the principal and the

agent have common beliefs on the distribution of θ which is assumed to be such that all expectations below exist.¹

The division of the payoff will be determined by a function which may depend on any variable which is observable by both parties, i.e., whose value is known by both parties after the action is taken and θ is realized. This assumption rules out contracts which are not incentive compatible. The function and its list of arguments, denoted $(S; z)$, will be called a *contract*. The value $S(z)$ is interpreted as the agent's share of the payoff while the principal's share is $x - S(z)$. The payoff, x , is assumed always to be observable by both parties. Whenever S depends on x , S is assumed to be continuously differentiable in x . In addition to x , the list, z , might include the action, a , provided, of course, that a is observable. If a is not observable, z might include an estimator, denoted α , of a . When available, the estimator will be called a *monitor*; i.e., a monitor is a random variable whose distribution is conditional on a . The class of available monitors is referred to as the *monitoring technology*, and denoted T .

Associated with the agent is a utility function, U^A , whose first argument is the agent's share of the payoff and whose second argument is the action taken by the agent. The agent is assumed to prefer less action to more action, other things equal.² We therefore assume that

$$U_1^A > 0; \quad U_2^A < 0 \quad (2)$$

and U^A is continuously differentiable. We will often assume that A is risk averse, i.e., that U^A is strictly concave in the first argument. Given a contract $(S; z)$, in Model 1 the agent chooses his action by solving the following maximization problem:

$$\max_{a \in A} V^A(S, a) \equiv E_\theta U^A(S(z), a). \quad (3)$$

Note that V^A depends on the functional form of S . The solution of this problem, given $(S; z)$, is denoted $a(S; z)$ or simply $a(S)$ when z is clear from the context.

¹ The reason for including this element of uncertainty will be discussed below. The interesting but more difficult case in which the two parties have different distributions on θ is not considered.

² This assumption distinguishes our analysis of moral hazard from those of Ross [10, 11], Leland [9], and Wilson [17]. As mentioned in the Introduction, these studies assume that the agent's action enters his utility only through the payoff; i.e., the action only determines the distribution of returns. In this case, for certain types of utility function which exhibit risk aversion for both agents, Pareto-optimal contracts exist in which no monitoring of the action or state of nature is required. These results can be compared with our Proposition 4 in Section 3, below.

In Model 2, the agent chooses a to solve

$$\max_{a \in A} U^A(S(z), a). \quad (3')$$

The solution of this problem, given $(S; z)$ and θ , is denoted $a[(S; z), \theta]$ or simply $a(S, \theta)$ when z is clear from the context.³ It is convenient to adopt the following notation:

$$V^A(S, a) = E_\theta U^A[S(z), a(\theta)] \quad (3'')$$

for any contract $(S; z)$ and any function $a(\cdot)$ with values in A . Although V^A as defined in (3'') is notationally identical to V^A as defined in (3), here the second argument of V^A is understood to be a function, whereas in (3), it is an element of A . We assume that the decision not to engage in a contract is always feasible and denote by $V^A(0, 0)$ the utility of A in this case.

Associated with the principal is a utility function, U^P , which is a function only of his share of the payoff. We assume U^P is monotone increasing, concave (i.e., P is either risk neutral or risk averse), and continuously differentiable. Given a contract, $(S; z)$, define

$$V^P(S, a) \equiv E_\theta U^P(X(a, \theta) - S(z)). \quad (4)$$

Note that V^P depends on the functional form of S .

The problem will be to discover contracts with the property that, given the availability of information, neither agent's expected utility can be increased without decreasing the expected utility of the other agent; i.e., Pareto-optimal contracts are sought. The qualification, "given the availability of information," is important since the existence of information will affect the nature of the optimal contract. In particular, we will be interested in the effects of making available a class of monitors, T , as mentioned above. Note that the Pareto-optimal contract will, in general, involve a *specification of a monitor* from the monitoring technology. The contract also specifies the share of each agent as a function both of the payoff and the specified monitor.

In the next section, we state and prove some results regarding the optimal contracts under various information structures.

³ A solution to (3) exists since V^A is continuous and A is compact; however, in general, $a(S; z)$ may be a correspondence; i.e., more than one action may solve (3). To avoid this and ensure that Pareto-optimality is well defined, we assume that if more than one action solves (3), $a(S; z)$ is the one which yields the highest expected utility for P . Similarly a solution to (3') exists and is continuous in θ since U^A is continuous in a and A is compact. In general $a[(S; z), \theta]$ may be a correspondence. In this case, we adopt the same convention as in (3).

3. PARETO OPTIMAL CONTRACTS AND POTENTIAL GAINS TO MONITORING

In this section, we (a) characterize the Pareto optimal contracts under various assumptions concerning the availability of perfect information and (b) identify conditions under which there are potential gains to monitoring as well as conditions under which there are no such gains.

First, consider the case where the realization of θ is observable. Since x is also observable, a can be inferred *ex post*. Therefore there are no gains to direct acquisition of information regarding a . This result is established formally in

PROPOSITION 1. *Let $(S_1 ; x, a, \theta)$ be any contract. Then there is a contract $(S_2 ; x, \theta)$ such that*

$$V^i(S_2, a(S_2)) \geq V^i(S_1, a(S_1)) \quad \text{for } i = A, P;$$

i.e., any contract which depends on $x, a,$ and θ can be dominated by one which depends on x and θ . This result holds for both Models 1 and 2. (The result for Model 1 was shown by Spence and Zeckhauser [13, f.n. 3, pp. 382–383] using a somewhat different proof.)

Proof. We first prove the proposition for Model 1. Let $a_1 = a(S_1)$, $X^*(\theta) = X(a_1, \theta)$ and define $(S_2 ; x, \theta)$ by

$$S_2(x, \theta) = S_1(X^*(\theta), a_1, \theta) - X^*(\theta) + x.$$

Let $a_2 = a(S_2)$. Hence

$$\begin{aligned} V^A(S_2, a_2) &\geq V^A(S_2, a_1) && \text{by definition of } a_2 \\ &= V^A(S_1, a_1) && \text{by construction of } S_2. \end{aligned}$$

Also,

$$\begin{aligned} X(a_2, \theta) - S_2(X(a_2, \theta), \theta) &= X(a_2, \theta) - S_1(X^*(\theta), a_1, \theta) + X^*(\theta) - X(a_2, \theta) \\ &= X(a_1, \theta) - S_1(X(a_1, \theta), a_1, \theta). \end{aligned}$$

Thus,

$$V^P(S_2, a_2) = V^P(S_1, a_1).$$

For Model 2, the only modifications required are to replace the symbols a_1 and a_2 by $a_1(\theta)$ and $a_2(\theta)$, respectively (except where a_1 and a_2 appear as arguments of V^A or V^P), where $a_1(\theta) = a(S_1, \theta)$ and $a_2(\theta) = a(S_2, \theta)$. Q.E.D.

The next proposition characterizes the form of the Pareto-optimal contracts which depend only on the payoff and the state. This contract specifies a “standard” payoff contingent on the state, $X^*(\theta)$. The agent receives an amount which depends on the standard payoff and, perhaps, on the state

plus the difference between the actual payoff and the standard. The principal receives an amount which depends only on the state and is therefore unaffected by the agent's choice of action.

PROPOSITION 2. *Any Pareto-optimal contract in the class of contracts depending only on x and θ is equivalent to (i.e., generates the same expected utilities as) a contract of the form*

$$S^*(x, \theta) = S_1(X^*(\theta), \theta) + x - X^*(\theta)$$

for some functions $S_1(\cdot, \cdot)$ and $X^*(\cdot)$. This result holds for both Models 1 and 2.

Proof. We first prove the proposition for Model 1. Suppose $(S_1; x, \theta)$ is Pareto-optimal.

Define

$$S^*(x, \theta) = S_1(X^*(\theta), \theta) + x - X^*(\theta),$$

where

$$X^*(\theta) = X(a_1, \theta) \quad \text{and} \quad a_1 = a(S_1).$$

Now, if $a^* = a(S^*)$,

$$\begin{aligned} V^P(S^*, a^*) &= E_\theta U^P[X(a^*, \theta) - S_1(X^*(\theta), \theta) - X(a^*, \theta) + X^*(\theta)] \\ &= E_\theta U^P[X(a_1, \theta) - S_1(X(a_1, \theta), \theta)] \\ &= V^P(S_1, a_1) \end{aligned}$$

and

$$\begin{aligned} V^A(S^*, a^*) &\geq V^A(S^*, a_1) = E_\theta U^A(S_1(X^*(\theta), \theta), a_1) \\ &= V^A(S_1, a_1). \end{aligned}$$

Since S_1 is Pareto-optimal, the above weak inequality must hold as an equality.

For Model 2, the proof is the same, except here we define

$$X^*(\theta) = X(a_1(\theta), \theta),$$

where $a_1(\theta) = a(S_1, \theta)$.

Q.E.D.

Next, consider the case in which the agent is risk neutral. With respect to the optimal sharing of risk, and ignoring incentive problems, it is well known that the optimal contract will result in the share of P being independent of θ . In the proof of the following proposition, we show that this result extends to the case in which incentive problems are considered. As a consequence, we show that the observability of a and θ would not lead to a Pareto superior contract.⁴

⁴ We are indebted to Ed Prescott for suggesting this point.

PROPOSITION 3. *If the agent is risk neutral, any contract which depends on x , a , and θ can be dominated by a contract which depends only on x . This result holds for Model 1. With the additional assumption that U^A is separable, i.e., $U^A(p, q) = p + g^A(q)$ for some function g^A , the result also holds for Model 2.*

Proof. We first prove the proposition for Model 1. Let $(S_1; x, a, \theta)$ be any contract with $a_1 = a(S_1)$. Define

$$(i) \quad c = E_\theta\{X(a_1, \theta) - S_1[X(a_1, \theta), a_1, \theta]\}$$

and let $(S_2; x)$ be the contract with

$$(ii) \quad S_2(x) = x - c.$$

Let $a_2 = a(S_2)$. Now

$$\begin{aligned} V^P(S_2, a_2) &= E_\theta U^P[X(a_2, \theta) - S_2(X(a_2, \theta))] \\ &= E_\theta U^P(c) \quad \text{by (ii)} \\ &= U^P[E_\theta\{X(a_1, \theta) - S_1[X(a_1, \theta), a_1, \theta]\}] \\ &\geq E_\theta U^P[X(a_1, \theta) - S_1[X(a_1, \theta), a_1, \theta]] \\ &= V^P(S_1, a_1). \end{aligned}$$

Thus P does not prefer S_1 to S_2 . For A , we have

$$\begin{aligned} V^A(S_2, a_2) &\geq V^A(S_2, a_1) \quad \text{by definition of } a_2 \\ &= E_\theta U^A[X(a_1, \theta) - c, a_1] \\ &= U^A[E_\theta\{X(a_1, \theta) - c\}, a_1] \quad \text{by risk neutrality} \\ &= U^A[E_\theta S_1(X(a_1, \theta), a_1, \theta), a_1] \quad \text{using (i)} \\ &= V^A(S_1, a_1) \quad \text{by risk neutrality.} \end{aligned}$$

This proves the result for Model 1. The proof for Model 2 is identical except that a_1 and a_2 are replaced by $a_1(\theta)$ and $a_2(\theta)$, respectively. Separability of U^A is required in showing that $V^A(S_2, a_2) \geq V^A(S_1, a_1)$. Q.E.D.

Propositions 1 and 3 identify two cases in which Pareto-optimal contracts will not involve the action. In Proposition 4, it is shown that these two situations are the only ones in which the action will not be included, i.e., whenever θ is unobservable and A is risk averse, optimal contracts will depend on the action if it is observable. Before proving the proposition, we require two preliminary results. The first is an adaptation of the general result that when marginal rates of substitution differ between two individuals, an allocation preferred by both is possible. The second establishes that for any contract depending only on x there is an action, a^* [$a^*(\theta)$ for Model 2], and another contract depending only on x , which both P and A would prefer to the original contract and

associated optimal action of the agent. Using this second result, Proposition 4 exhibits a contract which depends on both x and a (and θ for Model 2) and achieves an allocation which is Pareto superior to one achieved by a contract which depends only on x .

LEMMA 1. Let $v^i(s, a)$ for $i = A, P$ be two continuously differentiable functions for s in $[0, 1)$ and a in A . If, for some \bar{a} in $\text{Int } A$ and some \bar{s} , $v_1^A(\bar{s}, \bar{a}) > 0$, $v_2^A(\bar{s}, \bar{a}) = 0$, $v_1^P(\bar{s}, \bar{a}) < 0$, and $v_2^P(\bar{s}, \bar{a}) > 0$, then there exists (s^*, a^*) with s^* in $(0, 1)$ such that $v^i(s^*, a^*) > v^i(\bar{s}, \bar{a})$, $i = A, P$.

Proof. By the above inequalities $\text{grad } v^A(\bar{s}, \bar{a})$ and $\text{grad } v^P(\bar{s}, \bar{a})$ are linearly independent. Therefore there exists a vector $y = (y_1, y_2)$ in R^2 with $y_1 > 0$ such that

$$y \cdot \text{grad } v^i(\bar{s}, \bar{a}) > 0 \quad \text{for } i = A, P.$$

Now, by Taylor's theorem, there is an $\epsilon > 0$ such that for all (s, a) closer to (\bar{s}, \bar{a}) than ϵ

$$v^i(s, a) = v^i(\bar{s}, \bar{a}) + (s - \bar{s}, a - \bar{a}) \cdot \text{grad } v^i(\bar{s}, \bar{a}) + R^i,$$

where R^i approaches zero as (s, a) approaches (\bar{s}, \bar{a}) faster than $(s - \bar{s}, a - \bar{a}) \cdot \text{grad } v^i$, for $i = A, P$.

Therefore we can choose $\lambda > 0$ and sufficiently small such that if $(s, a) = (\bar{s}, \bar{a}) + \lambda y$, then

$$\begin{aligned} (s - \bar{s}, a - \bar{a}) \cdot \text{grad } v^i(\bar{s}, \bar{a}) + R^i &= \lambda y \cdot \text{grad } v^i(\bar{s}, \bar{a}) + R^i \\ &> 0 \quad \text{for } i = A, P \end{aligned}$$

since $y \cdot \text{grad } v^i(\bar{s}, \bar{a}) > 0$, for $i = A, P$. Let

$$(s^*, a^*) = (\bar{s}, \bar{a}) + \lambda y.$$

Then $v^i(s^*, a^*) > v^i(\bar{s}, \bar{a})$ for $i = A, P$. Since $y_1 > 0$, $s^* > \bar{s} \geq 0$. Q.E.D.

The proof of Lemma 2 is based on the observation that the marginal rates of substitution between the agent's action and the payoff are different for P and A if A chooses his action to solve problem (3) or (3'). The lemma holds for both models. To illustrate the proof for Model 1, consider the case in which the contract is of the form $S(x) = sx$ where $0 \leq s < 1$. The expected utility of both P and A can then be expressed as functions of s and a . Figure 1 depicts families of indifference curves for these two functions. Higher indifference curves of A represent higher levels of expected utility. The expected utility of P increases toward the southeast. When the contract is $\bar{S} = \bar{s}x$, then $a(\bar{S}) = \bar{a}$. At the point (\bar{a}, \bar{s}) the indifference curve of

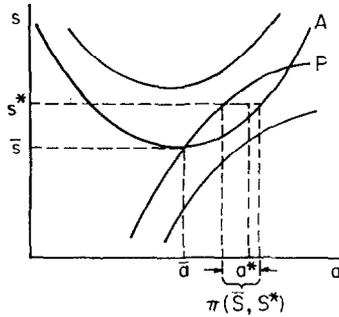


FIG. 1. Illustration of Lemma 2.

A is flat while that of P is upward sloping. Hence there is a point (a^*, s^*) which both P and A would prefer. This argument is generalized and made precise in Lemma 2.

LEMMA 2. (a) *Model 1: Suppose A is risk averse. Let $(\bar{S}; x)$ be a contract such that $\bar{a} = a(\bar{S})$ in Int A. Then there is a contract $(S^*; x)$ and a point a^* in A such that*

$$V^i(S^*, a^*) > V^i(\bar{S}, \bar{a}) \quad \text{for } i = A, P.$$

(b) *Model 2: Suppose U^A is separable and A is risk averse, i.e., $U^A(p, q) = u^A(p) + g^A(q)$, where u^A is strictly concave. Let $(\bar{S}; x)$ be a contract such that $\bar{a}(\theta) = a(\bar{S}, \theta)$ in Int A for all θ . Then there is a contract $(S^*; x)$ and a function $a^*(\theta)$, taking values in A, such that*

$$V^i(S^*, a^*) > V^i(\bar{S}, \bar{a}) \quad \text{for } i = A, P.$$

Proof. (a) *Model 1: We separate the proof into two cases.*

Case 1. $V_2^P(\bar{S}, \bar{a}) > 0$. Define $\bar{S}_s(x) = \bar{S}(x) + s$ for s in $[0, 1)$. Let $v^i(s, \bar{a}) = V^i(\bar{S}_s, \bar{a})$ for $i = A, P$. By definition of \bar{a} , we have

$$\begin{aligned} v_1^A(0, \bar{a}) &= E_\theta U_1^A[\bar{S}(X(\bar{a}, \theta)), \bar{a}] > 0, \\ v_2^A(0, \bar{a}) &= 0 \quad \text{by definition of } \bar{a}, \\ v_1^P(0, \bar{a}) &= -E_\theta [U^{P'}(X(\bar{a}, \theta) - \bar{S}(X(\bar{a}, \theta)))] < 0, \\ v_2^P(0, \bar{a}) &= V_2^P(\bar{S}, \bar{a}) > 0. \end{aligned}$$

Therefore, by Lemma 1, there is an s^* in $(0, 1)$ and a^* in A such that

$$v^i(s^*, a^*) > v^i(0, \bar{a}) \quad \text{for } i = A, P.$$

Let $S^*(x) = \bar{S}(x) + s^*$. Then, by definition of v^i , $V^i(S^*, a^*) = v^i(s^*, a^*) > v^i(0, \bar{a}) = V^i(\bar{S}, \bar{a})$ for $i = A, P$. This establishes the lemma for Case 1.

Case 2. $V_2^P(\bar{S}, \bar{a}) \leq 0$. Consider the problem

$$\max_S V^P(S, \bar{a})$$

subject to

$$V^A(S, \bar{a}) \geq V^A(\bar{S}, \bar{a}),$$

where $(S; x)$ is a contract. Clearly, any solution, S , to this problem has

$$V^i(S, \bar{a}) \geq V^i(\bar{S}, \bar{a}), \quad \text{for } i = A, P.$$

From the Euler equation, a necessary condition that S solve the problem is

$$U^{P'}(X(\bar{a}, \theta) - S(X(\bar{a}, \theta))) = \varphi U_1^A(S(X(\bar{a}, \theta)), \bar{a}) \quad \text{for all } \theta,$$

where φ is a Lagrange multiplier. Note that $\varphi = U^{P'}/U_1^A > 0$.

Differentiating this expression with respect to θ gives

$$\varphi U_{11}^A S' X_2 = U^{P''}(1 - S') X_2$$

or

$$S' = U^{P''}/[U^{P''} + \varphi U_{11}^A].$$

Since $U_{11}^A < 0$, $U^{P''} \leq 0$, and $\varphi > 0$, $0 \leq S' < 1$ for all θ .

Thus we may find a contract $(S; x)$ with $0 \leq S' < 1$ for all θ and such that $V^i(S, \bar{a}) \geq V^i(\bar{S}, \bar{a})$ for $i = A, P$. Now if $0 \leq S' < 1$, then

$$\begin{aligned} V_2^P(S, \bar{a}) &= E_\theta[U^{P'}(X - S(X)) X_1(\bar{a}, \theta)(1 - S'(X))] \\ &> 0, \end{aligned}$$

where X is evaluated at (\bar{a}, θ) . We may now apply the results of Case 1 to $[(S; x), \bar{a}]$, thus completing the proof for Model 1.

(b) Model 2. The proof is similar to part (a). Define \bar{S}_s as in part (a) and let

$$\bar{a}_t(\theta) = \bar{a}(\theta) + t \quad \text{for } 0 \leq t < t_0,$$

where $t_0 > 0$ is chosen so that $\bar{a}_t(\theta) \in A$ for $0 \leq t < t_0$ and all θ (such a t_0 can be chosen since $\bar{a}(\theta) \in \text{Int } A$ for all θ). Define

$$v^i(s, t) = V^i(\bar{S}_s, \bar{a}_t) \quad \text{for } i = A, P.$$

Case 1. $v_2^P(0, 0) > 0$. In this case, we have

$$\begin{aligned} v_1^A(0, 0) &= E_\theta U_1^A[\bar{S}(X(\bar{a}(\theta), \theta)), \bar{a}(\theta)] > 0, \\ v_2^A(0, 0) &= 0 \quad \text{by definition of } \bar{a}(\theta), \\ v_1^P(0, 0) &= -E_\theta U^{P'}[X(\bar{a}(\theta), \theta) - \bar{S}(X(\bar{a}(\theta), \theta))] < 0, \\ v_2^P(0, 0) &> 0 \quad \text{by assumption.} \end{aligned}$$

Therefore, by Lemma 1, there is an $s^* > 0$ and $t^* \in (0, t_0)$ such that

$$v^i(s^*, t^*) > v^i(0, 0) \quad \text{for } i = A, P.$$

Let $S^*(x) = \bar{S}(x) + s^*$ and $a^*(\theta) = \bar{a}(\theta) + t^*$. Then by definition of v^i ,

$$V^i(S^*, a^*) = v^i(s^*, t^*) > v^i(0, 0) = V^i(\bar{S}, \bar{a}) \quad \text{for } i = A, P.$$

Case 2. $v_2^P(0, 0) \leq 0$. Consider the problem

$$\max_S V^P(S, \bar{a})$$

subject to

$$V^A(S, \bar{a}) \geq V^A(\bar{S}, \bar{a}),$$

where $(S; x)$ is a contract. Clearly, any solution to this problem has $V^i(S, \bar{a}) \geq V^i(\bar{S}, \bar{a})$ for $i = A, P$. From the Euler equation, a necessary condition that S solve the problem is

$$U^{P'}[X(\bar{a}(\theta), \theta) - S(X(\bar{a}(\theta), \theta))] = \varphi U_1^A[S(X(\bar{a}(\theta), \theta)), \bar{a}(\theta)] \quad \text{for all } \theta,$$

where $\varphi > 0$ is a Lagrange multiplier, as in the proof for Model 1.

Differentiating this expression with respect to θ and recalling that U^A is separable gives

$$\varphi U_{11}^A S' = U^{P''}(1 - S')$$

or

$$S' = U^{P''}/[U^{P''} + \varphi U_{11}^A].$$

Since $U_{11}^A < 0$, $U^{P''} \leq 0$, we have $0 \leq S' < 1$ for all θ .

Thus we may find a contract $(S; x)$ with $0 \leq S' < 1$ for all θ and such that $V^i(S, \bar{a}) \geq V^i(\bar{S}, \bar{a})$ for $i = A, P$. Now if we redefine $\bar{S}_s(x) = S(x) + s$, we have

$$\begin{aligned} v_2^P(0, 0) &= E_\theta U^{P'}[X - S(X)] X_1(1 - S'(X)) \\ &> 0 \quad \text{since } 0 \leq S' < 1, \end{aligned}$$

where X and X_1 are evaluated at $(\bar{a}(\theta), \theta)$. We may now apply the results of Case 1 to $(S; x)$ and $\bar{a}(\theta)$. This completes the proof for Model 2. Q.E.D.

COROLLARY 1. Let $(S; x)$ and $a \in A(\theta)$ for Model 2) be any contract and action.

(a) If P is risk neutral and $S(x) \neq \text{constant}$, then there is a contract $(S^*; x)$ such that $S^*(x) = \text{constant}$, $V^i[(S^*; x), a] > V^i[(S; x), a]$, for $i = A, P$.

(b) If P is risk averse, and $S'(x) \notin (0, 1)$, then there is a contract $(S^*; x)$ such that $S^{*'}(x) \in (0, 1)$ and the above inequalities are satisfied.

These results apply to both models. For Model 2, the separability of U^A is assumed.

Proof. Denote by S^* the solution to

$$\text{Max}_S V^P(\bar{S}, a)$$

subject to $V^A(\bar{S}, a) \geq V^A(S, a)$. As shown in the proof of Case 2, lemma 2

$$S^{*'} = \frac{U^{P''}}{U^{P''} + \varphi U_{11}^A} \quad \text{for all } x.$$

If $U^{P''} = 0$ then $S^{*'} = 0$. If $U^{P''} < 0$ then $0 < S^{*'} < 1$. If S' does not satisfy these conditions then the necessary conditions for an optimum are not satisfied and therefore (S^*, a) dominates (S, a) . Q.E.D.

A second corollary to Lemma 2 will be required in Section 4. Before proving this corollary, we introduce some useful notation. For any pair of contracts $(S^0; x), (S^1; x)$, define

$$\pi(S^0, S^1) = \{a \in A: V^i(S^1, a) > V^i(S^0, a_0) \quad \text{for } i = A, P\},$$

where $a_0 = a(S_0)$ (see Fig. 1).

COROLLARY 2. If $(\bar{S}; x)$ is such that $a(\bar{S}) \in \text{Int } A$ and A is risk averse, then there is $(S^*; x)$ such that $\text{Int } \pi(\bar{S}, S^*) \neq \emptyset$. This corollary applies only to Model 1.

Proof. By Lemma 2, there exists a contract $(S^*; x)$ and a^* in A such that $a^* \in \pi(\bar{S}, S^*)$. Furthermore, it is clear from the proof of Lemma 2, that if \bar{S} is continuous, S^* can be chosen to be continuous also. By continuity of V^i for $i = A, P$ and S^* , there is a neighborhood of a^* on which $V^i(S^*, a) > V^i(\bar{S}, \bar{a})$ for $i = A, P$. Therefore $\text{Int } \pi(\bar{S}, S^*) \neq \emptyset$. Q.E.D.

PROPOSITION 4. (a) Model 1: Let $(S_1; x)$ be any contract such that $a_1 = a(S_1)$ is in $\text{Int } A$. If A is risk averse, then there is a contract $(S_2; x, a)$ such that

$$V^i(S_2, a_2) > V^i(S_1, a_1) \quad \text{for } i = A, P,$$

where $a_2 = a(S_2)$; i.e., any contract depending only on x can be strictly dominated by one depending on x and a .

(b) *Model 2.* Let $(S_1; x)$ be any contract such that $a_1(\theta) = a(S_1, \theta)$ is in $\text{Int } A$ for all θ . If U^A is separable and A is risk averse, then there is a contract $(S_2; x, a, \theta)$ such that

$$V^i(S_2, a_2) > V^i(S_1, a_1) \quad \text{for } i = A, P,$$

where $a_2(\theta) = a(S_2, \theta)$. Moreover, if X is one-to-one in θ for any a , then $(S_2; x, a, \theta)$ can be replaced by $(S_3; x, a)$.

Proof. (a) *Model 1:* By Lemma 2, there is a contract $(S^*; x)$ and an action a^* in A such that

$$V^i(S^*, a^*) > V^i(S_1, a_1) \quad \text{for } i = A, P.$$

Let

$$S_2(x, a) = \begin{cases} S^*(x) & \text{if } a = a^* \\ 0 & \text{if } a \neq a^*. \end{cases}$$

Then

$$V^A(S^*, a^*) > V^A(S_1, a_1) \geq V^A(0, 0) \geq V^A(0, a)$$

for all a in A since $a_1 > 0$. Thus $a_2 = a^*$. Since $S_2(x, a^*) = S^*(x)$,

$$V^i(S_2, a_2) = V^i(S^*, a^*) > V^i(S_1, a_1) \quad \text{for } i = A, P.$$

(b) *Model 2:* The proof is similar to part (a). Define $(S_1; x, a, \theta)$ by

$$S_2(x, a, \theta) = \begin{cases} S^*(x) & \text{if } a = a^*(\theta) \\ 0 & \text{otherwise,} \end{cases}$$

where $(S^*; x)$ and $a^*(\theta)$ are defined as in part (a) using Lemma 2. Clearly, $a_1(\theta) \equiv a(S_2, \theta) = a^*(\theta)$ for any θ . Therefore,

$$V^i(S_2, a_2) = V^i(S^*, a^*) > V^i(S_1, a_1) \quad \text{for } i = A, P.$$

If X is one-to-one in θ , define $\bar{\theta}(x, a)$ to be the unique value of θ such that $X(a, \theta) = x$. Then define $(S_3; x, a)$ by

$$S_3(x, a) = S_2[x, a, \bar{\theta}(x, a)]. \quad \text{Q.E.D.}$$

The contract between P and A can be thought of as arranging for the trade of three "goods": "risky return," "nonrisky return," and "agent's action." Proposition 4 asserts, essentially, that if "agent's action" can be

traded, an allocation can be achieved which is Pareto-superior to any allocation made when the action cannot be traded.

In the next proposition, we show that the type of contract used in the proof of Proposition 4 is Pareto-optimal when both x and a (and θ in Model 2) are observable. These contracts have the property that the agent's share depends only on the payoff, provided he takes a specific action. Thus, in the "goods" interpretation, the optimal arrangement is shown to be one in which an action is agreed on in return for certain amounts of risky and nonrisky return. The conditions for optimal risk sharing are given in Corollary 1.

PROPOSITION 5. (a) *Model 1: Any contract $(S_1; x, a)$ is equivalent to (i.e., generates the same expected utilities as) a contract $(S^*; x, a)$ of the form*

$$S^*(x, a) = \begin{cases} \bar{S}(x) & \text{for } a = a^* \\ 0 & \text{for } a \neq a^*, \end{cases}$$

for some $a^* \in A$ and function \bar{S} . Thus contracts of this form are Pareto-optimal in the class of contracts depending on x and a .⁵

(b) *Model 2: Any contract $(S_1; x, a, \theta)$ is equivalent to a contract $(S^*; x, a, \theta)$ of the form*

$$S^*(x, a, \theta) = \begin{cases} \bar{S}(x) & \text{for } a = a^*(\theta) \\ 0 & \text{for } a \neq a^*(\theta), \end{cases}$$

for some function $a^*(\theta)$ taking values in A and some function \bar{S} . Thus contracts of this form are Pareto-optimal in the class of contracts depending on x, a , and θ .

Proof. (a) Model 1: Let $(S_1; x, a)$ be any contract with $a_1 = a(S_1)$. Define

$$S^*(x, a) = \begin{cases} S_1(x, a_1) & \text{if } a = a_1 \\ 0 & \text{if } a \neq a_1. \end{cases}$$

From the proof of Proposition 4, $a^* \equiv a(S^*) = a_1$. Therefore $V^i(S^*, a^*) = V^i(S_1, a_1)$ for $i = A, P$.

(b) Model 2: The proof is identical. Q.E.D.

COROLLARY 3. *Subject to the assumptions of Proposition 4, any contract which depends only on the payoff can be strictly dominated by a contract which depends only on the payoff and the state and is of the form given in Proposition 2. This result holds for both Models 1 and 2.*

⁵ Spence and Zeckhauser [13, p. 382] state a similar result, but without a formal proof.

Proof. By Proposition 4, any contract depending on x can be strictly dominated by one depending on x , and a (and θ for Model 2). But by Proposition 1, such a contract can be dominated by a contract which depends only on x and θ . By Proposition 2, this contract can be dominated by one of the given form. Q.E.D.

Thus far we have studied the structure of Pareto-optimal contracts under various assumptions regarding perfect information on the agent's action and the random state. The results obtained also have implications for the potential usefulness of imperfect monitors of the agent's action in Model 1. These results show that if θ is unobservable and A is risk averse, then both P and A would prefer a situation in which the agent could be perfectly supervised at no cost to one in which information about A 's action is unavailable.⁶ Thus, in this case there are *potential* gains to imperfect monitoring. Moreover, it is clear that when θ is observable and/or A is risk neutral, there are no gains to monitoring. From a methodological point of view, these results imply that it is not possible to simplify the analysis of monitoring by assuming away either exogenous uncertainty or risk aversion on the part of the agent. We summarize the implications of this section regarding imperfect monitoring in

PROPOSITION 6. *Necessary and sufficient conditions for potential gains to monitoring in Model 1 are*

- (a) θ is unobservable
- (b) A is risk averse.

Therefore, we assume in the next section that θ is not observable and A is risk averse.

4. IMPERFECT MONITORING

In this section, we explain the widespread use of certain types of monitoring contracts. In particular, we

- (a) adapt a proof of Shavell [12] to show that when there are potential gains to monitoring (i.e., under the conditions established in Section 3), the use of monitors allows realization of some of these gains (in Model 1) and
- (b) prove that the optimal use of a monitor (in Model 1) is via a *dichotomous* contract.

A dichotomous contract has two branches: If the results of monitoring reveal the action to be acceptable, the agent is paid according to a pre-

⁶ Stiglitz [14] and Spence and Zeckhauser [13] both mention this result, but without proof or qualification as to the risk neutrality of A (and in the case of Stiglitz, the observability of θ).

determined schedule; otherwise the agent receives a less preferred fixed payment (e.g., he is dismissed). In addition to explaining the observed use of dichotomous contracts, we also derive some further testable implications under more restrictive assumptions.

In general, even when there are gains to perfect information on a , it would appear that these gains may be impossible to realize through imperfect monitoring. The introduction of imperfect information on the agent's action into a contract produces two opposing effects on the welfare of the parties to the contract. First, since the information is imperfect, additional uncertainty is introduced. Since both the principal and agent are risk averse, this additional uncertainty tends to reduce welfare. Second, the inclusion of monitoring can motivate the agent to take an action which, neglecting the first effect, would make both parties better off. For example, if the only available monitors have distributions which are independent of a , then only the first effect will be present. In this extreme case, it is clear that monitoring is not worthwhile. The first result of this section shows that, other than this extreme case, monitoring is always beneficial. This result has been shown in a similar context by Shavell.⁷ Our adaptation of his proof is included mainly for completeness and as background for the proof of Proposition 8.

In this section we maintain the assumption that A is strictly risk averse and θ and a are unobservable so that there are potential gains to monitoring. We further assume that the monitoring technology contains at least one monitor of the form $\alpha = a + \delta$, where δ is a random variable with support $[-\alpha_0, \alpha_1]$ for some finite α_0, α_1 . Such monitors will be referred to as *regular monitors*.⁸ Finally, since X is arbitrary, we may assume, without loss of generality, that θ is uniform on $[0, 1]$.

PROPOSITION 7. *Let $(S^0; x)$ be any contract such that $a_0 = a(S^0)$ is in Int A . If α is any regular monitor, then there is a contract $(S^*; x, \alpha)$ such that*

$$E_\alpha |_{a^*} V^A(S^*, a^*) > V^A(S^0, a_0),$$

$$E_\alpha |_{a^*} V^P(S^*, a^*) > V^P(S^0, a_0),$$

where $a^* = a(S^*)$ and $E_\alpha |_{a^*}$ denotes the expectation with respect to the distribution of α conditional on the action being a^* .

Proof. Define

$$S^*(x, \alpha) = S^0(x) + \epsilon[\alpha - \bar{\alpha}(a_0)] + \beta,$$

⁷ This has also been shown by Holmstrom [7].

⁸ We assume that in this section, for every α in T , α and θ are independent. Thus we consider the possible benefits to monitoring using only monitors which are not correlated with the state of nature. In particular, this assumption excludes the use of output, x , as a monitor, although contracts may, of course, depend on x as in Section 3.

where ϵ and β are real constants, and $\bar{\alpha}(a_0) = E(\alpha | a = a_0)$. Note that at $\epsilon = \beta = 0$, $S^* \equiv S^0$. The proof consists of showing that there exists ϵ, β which make both A and P better off than at that S^0 . Let $W^A(\epsilon, \beta, a)$ be A 's expected utility under S^* when A 's action is a , i.e.,

$$W^A(\epsilon, \beta, a) = \int_{a-\alpha_0}^{a+\alpha_1} \int_0^1 U^A[S^*(X(a, \theta), t), a] f_\delta(t - a) d\theta dt,$$

where $\alpha = a + \delta$ and f_δ is the density of δ and $f_\delta > 0$ on $(-\alpha_0, \alpha_1)$. The optimal action $a^* = a(\epsilon, \beta)$ for S^* is given by

$$\begin{aligned} \frac{\partial}{\partial a} W^A(\epsilon, \beta, a^*) &= f_\delta(\alpha_1) \int_0^1 U^A[S^*(X, a^* + \alpha_1), a^*] d\theta \\ &\quad - f_\delta(-\alpha_0) \int_0^1 U^A[S^*(X, a^* - \alpha_0), a^*] d\theta \\ &\quad + \int_{a^*-\alpha}^{a^*+\alpha_1} \int_0^1 \{U_1^A S^0 X_1 + U_2^A\} f_\delta(t - a^*) \\ &\quad - U^A f'_\delta(t - a^*) d\theta dt \\ &= \int_{a^*-\alpha_0}^{a^*+\alpha_1} \int_0^1 [U_1^A S^0 X_1 + U_2^A] f_\delta(t - a^*) d\theta dt \\ &\quad + \int_{a^*-\alpha_0}^{a^*+\alpha_1} \int_0^1 U_1^A \epsilon f_\delta(t - a^*) d\theta dt \\ &= 0, \end{aligned}$$

where the second expression is obtained by integration by parts. Note that at $\epsilon = \beta = 0$, the above expression is the first-order condition for the action given the contract S^0 since $U_1^A S^0 X_1 + U_2^A$ does not depend on t . Therefore, $a(0, 0) = a_0$.

Let

$$v^A(\epsilon, \beta) = W^A(\epsilon, \beta, a(\epsilon, \beta)).$$

We now show that $v_1^A(0, 0) = 0$ and $v_2^A(0, 0) > 0$:

$$\begin{aligned} v_1^A(0, 0) &= W_1^A(0, 0, a_0) + W_3^A(0, 0, a_0) \frac{\partial a^*}{\partial \epsilon} \Big|_{\epsilon=\beta=0} \\ &= W_1^A(0, 0, a_0) \quad \text{since } W_3^A(\epsilon, \beta, a^*) = 0 \text{ for any} \\ &\quad \epsilon, \beta, \text{ and } a^* = a(\epsilon, \beta) \\ &= \int_{a_0-\alpha_0}^{a_0+\alpha_1} \int_0^1 U_1^A[S^0(X), a_0][t - \bar{\alpha}(a_0)] f_\delta(t - a_0) d\theta dt = 0, \end{aligned}$$

and

$$\begin{aligned} v_2^A(0, 0) &= W_2^A(0, 0, a_0) + W_3^A(0, 0, a_0) \frac{\partial a^*}{\partial \beta} \Big|_{\epsilon=\beta=0} \\ &= \int_{a_0-\alpha_0}^{a_0+\alpha_1} \int_0^1 U_1^A[S^0(X), a_0] f_\delta(t - a_0) d\theta dt > 0. \end{aligned}$$

Next let

$$\begin{aligned} W^P(\epsilon, \beta, a) &= \int_{a-\alpha_0}^{a+\alpha_1} \int_0^1 U^P[X - S^*(X, t)] f_\delta(t - a) d\theta dt, \\ v^P(\epsilon, \beta) &= W^P(\epsilon, \beta, a(\epsilon, \beta)). \end{aligned}$$

We now show that $v_1^P(0, 0) \neq 0$:

$$\begin{aligned} v_1^P(0, 0) &= W_1^P(0, 0, a_0) + W_3^P(0, 0, a_0) \frac{\partial a^*}{\partial \epsilon} \Big|_{\epsilon=\beta=0} \\ &= - \int_{a_0-\alpha_0}^{a_0+\alpha_1} \int_0^1 U^P[X - S^0(X)] [t - \bar{\alpha}(a_0)] f_\delta(t - a_0) d\theta dt \\ &\quad + W_3^P(0, 0, a_0) \frac{\partial a^*}{\partial \epsilon} \Big|_{\epsilon=\beta=0} \\ &= W_3^P(0, 0, a_0) \frac{\partial a^*}{\partial \epsilon} \Big|_{\epsilon=\beta=0} \end{aligned}$$

since $U^P[X - S^0(X)]$ does not depend on t and $E_{\alpha/a_0}[\alpha - \bar{\alpha}(a_0)] = 0$. To complete the proof that $v_1^P(0, 0) \neq 0$, we must show that $W_3^P(0, 0, a_0) \neq 0$ and $(\partial a^*/\partial \epsilon)|_{\epsilon=\beta=0} \neq 0$. To see that $W_3^P(0, 0, a_0) \neq 0$, note that without loss of generality, we may assume that S^0 and a_0 solve

$$\max_{S, a} \int_0^1 U^P[X - S(X)] d\theta$$

subject to

$$\begin{aligned} \int_0^1 U^A[S(X), a] d\theta &\geq K \\ \int_0^1 \{U_1^A[S(X), a] S'(X) X_1 + U_2^A[S(X), a]\} d\theta &= 0 \end{aligned}$$

for some K since otherwise we can find another contract depending only on x which does solve this problem and dominates S^0 , and we can substitute this

contract for S^0 in the proof. Using the first-order, necessary conditions for the above problem for a and S it can be shown that

$$\int_0^1 U^{P'}[X - S^0(X)][1 - S^{0'}(X)] X_1 d\theta \neq 0.$$

But, using integration by parts,

$$\begin{aligned} W_3^{P'}(0, 0, a_0) &= \int_{a_0 - \alpha_0}^{a_0 + \alpha_0} \int_0^1 U^{P'}[X - S^0(X)][1 - S^{0'}(X)] X_1 f_\delta(t - a_0) d\theta dt \\ &= \int_0^1 U^{P'}[X - S^0(X)][1 - S^{0'}(X)] X_1 d\theta \neq 0, \end{aligned}$$

It remains to show that $(\partial a^*/\partial \epsilon)|_{\epsilon=\beta=0} \neq 0$. Using the second order condition for a and the fact that $a(0, 0) = a_0$, we need only show that $(\partial^2/\partial a \partial \epsilon) W^A(0, 0, a_0) > 0$. But

$$\begin{aligned} \frac{\partial^2}{\partial a \partial \epsilon} W^A(0, 0, a_0) &= \int_{a_0 - \alpha_0}^{a_0 + \alpha_1} \int_0^1 \{U_{11}^A[S^0(X), a_0] S^{0'}(X) X_1 \\ &\quad + U_{21}^A[S^0(X), a_0]\} [t - \bar{\alpha}(a_0)] f_\delta(t - a_0) d\theta dt \\ &\quad + \int_{a_0 - \alpha_0}^{a_0 + \alpha_1} \int_0^1 U_1^A[S^0(X), a_0] f_\delta(t - a_0) d\theta dt. \end{aligned}$$

The first term is zero since $U_{11}^A S^{0'} X_1 + U_{21}^A$ is independent of t and $E_{\alpha|a_0}[\alpha - \bar{\alpha}(a_0)] = 0$. Since the second term is positive, we have proved the assertion.

To summarize the proof thus far, we have shown that $v_1^A(0, 0) = 0$, $v_2^A(0, 0) > 0$, $v_1^P(0, 0) \neq 0$. Therefore, $\text{grad } v^A(0, 0)$ and $\text{grad } v^P(0, 0)$ are linearly independent and by a proof similar to that of Lemma 1, it can be shown that there exists (ϵ, β) near $(0, 0)$ such that $v^i(\epsilon, \beta) > v^i(0, 0)$ for $i = A, P$. Since $v^i(\epsilon, \beta)$ is the expected utility of individual i under S^* and $v^i(0, 0)$ is the expected utility of agent i under S^0 , we are done. Q.E.D.

We have shown, thus far, that, under some relatively weak assumptions on the monitoring technology, it is always beneficial to incorporate monitoring. Our next result demonstrates that it is always optimal to use the monitor in a contract of the form

$$S(x, \alpha) = \begin{cases} S^0(x, \alpha) & \text{for } \alpha \geq \hat{\alpha} \\ w & \text{otherwise,} \end{cases}$$

where $(S^0; x, \alpha)$ is a continuous contract, w is a real constant for which $U^A(w, a)$ is defined, and $\hat{\alpha}$ is a real parameter. Contracts of this form will be referred to as *dichotomous* contracts. This contract can be interpreted as one in which the agent is paid according to some prespecified schedule (depending

on output and the monitoring result) if his action is judged to be acceptable based on the monitoring result. If the action is judged to be unacceptable, he receives a fixed and less preferred payment, w . Casual observation indicates that this type of contract is widely used in many areas. For example, in employment contracts, the lower branch can be interpreted as dismissal of the worker. In insurance, it can be interpreted as a refusal to pay the compensation or as a cancellation of the policy. The prevalence of such contracts is explained by

PROPOSITION 8. *Let $(S^0; x, \alpha)$ be any continuously differentiable contract with $a_0 = a(S^0)$ in $Int A$ and $\alpha = a + \delta$ a regular monitor with $f_\delta(-\alpha_0) = 0$ and $f'_\delta(-\alpha_0) > 0$. Then there is a dichotomous contract $(S^*; x, \alpha)$ such that*

$$E_{\alpha|a^*} V^A(S^*, a^*) > E_{\alpha|a_0} V^A(S^0, a_0),$$

$$E_{\alpha|a^*} V^P(S^*, a^*) > E_{\alpha|a_0} V^P(S^0, a_0),$$

where $a^* = a(S^*)$.

Proof. The proof is similar to that of Proposition 7. Define

$$S^*(x, \alpha) = \begin{cases} S^1(x, \alpha) = S^0(x, \alpha) + h(\epsilon) + \beta & \text{if } \alpha \geq a_0 - \alpha_0 + \epsilon \\ w & \text{otherwise,} \end{cases}$$

where $w < \min_{\theta \in [0, 1]} S^0[X(a_0, \theta), a_0 - \alpha_0], 0$, ϵ and β are real constants whose values are defined below, and h is a function with $h(0) = h'(0) = 0$ and $h'(\epsilon) > 0$ for all $\epsilon \neq 0$. Note that at $\epsilon = \beta = 0$ and $a = a_0$, $S^* \equiv S^0$. We first show that if $a^* = a(S^*) = a(\epsilon, \beta)$, then $a(0, 0) = a_0$. First note that A will always choose $a^* + \alpha_1 > a_0 - \alpha_0 + \epsilon$ since otherwise he would be better off in autarky, but for ϵ sufficiently small he prefers S^* to autarky. Let $W^A(\epsilon, \beta, a)$ be A 's expected utility under S^* if he chooses action a , i.e.

$$W^A(\epsilon, \beta, a) = \int_{a_0 - \alpha_0 + \epsilon}^{a + \alpha_1} \int_0^1 U^A[S^1(X, t), a] f_\delta(t - a) d\theta dt$$

$$+ F_\delta(a_0 - \alpha_0 + \epsilon - a) U^A(w, a).$$

The first-order condition for a^* is (using integration by parts)

$$W_3^A(\epsilon, \beta, a) = \int_{a_0 - \alpha_0 + \epsilon}^{a + \alpha_1} \int_0^1 \{ U_1^A[S^1(X, t), a][S_1^0(X, t) X_1 + S_2^0(X, t)]$$

$$+ U_2^A[S^1(X, t), a] f_\delta(t - a) d\theta dt$$

$$+ f_\delta(a_0 - \alpha_0 + \epsilon - a) \left\{ \int_0^1 U^A[S^1(X, a_0 - \alpha_0 + \epsilon), a] d\theta \right.$$

$$\left. - U^A(w, a) \right\}$$

$$+ F_\delta(a_0 - \alpha_0 + \epsilon - a) U_2^A(w, a) = 0.$$

Since $h(0) = 0$, the first term above is zero at $\epsilon = \beta = 0$, $a = a_0$ using the first-order condition for a_0 . Since $f_\delta(-\alpha_0) = F_\delta(-\alpha_0) = 0$, we see that $W_3^A(0, 0, a_0) = 0$. Thus, $a(0, 0) = a_0$. Let

$$v^A(\epsilon, \beta) = W^A(\epsilon, \beta, a(\epsilon, \beta)).$$

We now show, as in Proposition 7, that $v_1^A(0, 0) = 0$, and $v_2^A(0, 0) > 0$:

$$\begin{aligned} v_1^A(0, 0) &= W_1^A(0, 0, a_0) \quad \text{since } W_3^A(0, 0, a_0) = 0 \\ &= f_\delta(-\alpha_0) \left[U^A(w, a_0) - \int_0^1 U^A[S^0(X, a_0 - \alpha_0), a_0] d\theta \right] \\ &\quad + h'(0) \int_{a_0 - \alpha_0}^{a_0 + \alpha_1} \int_0^1 U_1^A[S^0(X, t), a_0] f_\delta(t - a_0) d\theta dt \\ &= 0 \end{aligned}$$

since $f_\delta(-\alpha_0) = h'(0) = 0$, and

$$\begin{aligned} v_2^A(0, 0) &= W_2^A(0, 0, a_0) = \int_{a_0 - \alpha_0}^{a_0 + \alpha_1} \int_0^1 U_1^A[S^0(X, t), a_0] f_\delta(t - a_0) d\theta dt \\ &> 0. \end{aligned}$$

Next, let

$$\begin{aligned} W^P(\epsilon, \beta, a) &= \int_{a_0 - \alpha_0 + \epsilon}^{a_0 + \alpha_1} \int_0^1 U^P[X - S^1(X, t)] f_\delta(t - a) d\theta dt \\ &\quad + F_\delta(a_0 - \alpha_0 + \epsilon - a) \int_0^1 U^P(X - w) d\theta, \\ v^P(\epsilon, \beta) &= W^P(\epsilon, \beta, a(\epsilon, \beta)). \end{aligned}$$

We now show that $v_1^P(0, 0) \neq 0$:

$$\begin{aligned} v_1^P(0, 0) &= W_1^P(0, 0, a_0) + W_3^P(0, 0, a_0) \left. \frac{\partial a^*}{\partial \epsilon} \right|_{\epsilon=\beta=0} \\ &= f_\delta(-\alpha_0) \left[\int_0^1 \{ U^P(X - w) - U^P[X - S^0(x, a_0 - \alpha_0)] \} d\theta \right] \\ &\quad - h'(0) \int_{a_0 - \alpha_0}^{a_0 + \alpha_1} \int_0^1 U^P[X - S^0(X, t)] f_\delta(t - a_0) d\theta dt \\ &\quad + W_3^P(0, 0, a_0) \left. \frac{\partial a^*}{\partial \epsilon} \right|_{\epsilon=\beta=0} \\ &= W_3^P(0, 0, a_0) \left. \frac{\partial a^*}{\partial \epsilon} \right|_{\epsilon=\beta=0}. \end{aligned}$$

To complete the proof that $v_1^A(0, 0) \neq 0$, we must show that $W_3^P(0, 0, a_0) \neq 0$ and $(\partial a^*/\partial \epsilon)|_{\epsilon=\beta=0} \neq 0$. To see that $W_3^P(0, 0, a_0) \neq 0$, note that, as in Proposition 7, without loss of generality, we may assume that S^0 and a_0 solve

$$\max_{S, a} \int_{a-\alpha_0}^{a+\alpha_1} \int_0^1 U^P[X - S(X, t)] f_\delta(t - a) d\theta dt$$

subject to

$$\int_{a-\alpha_0}^{a+\alpha_1} \int_0^1 U^A[S(X, t), a] f_\delta(t - a) d\theta dt \geq K$$

$$\frac{\partial}{\partial a} \int_{a-\alpha_0}^{a+\alpha_1} \int_0^1 U^A[S(X, t), a] f_\delta(t - a) d\theta dt = 0.$$

The first-order condition for a at $a = a_0$ is

$$\begin{aligned} & \frac{\partial}{\partial a} \int_{a_0-\alpha_0}^{a_0+\alpha_1} \int_0^1 U^P[X - S(X, t)] f_\delta(t - a_0) d\theta dt \\ & + \lambda_1 \frac{\partial}{\partial a} \int_{a_0-\alpha_0}^{a_0+\alpha_1} \int_0^1 U^A[S(X, t), a] f_\delta(t - a) d\theta dt \\ & - \lambda_2 \frac{\partial^2}{\partial a^2} \int_{a_0-\alpha_0}^{a_0+\alpha_1} \int_0^1 U^A[S(X, t), a] f_\delta(t - a) d\theta dt = 0, \quad (i) \end{aligned}$$

where $\lambda_1 \geq 0$ and λ_2 are Lagrange multipliers.

We wish to show that the first term of (i) is not zero. Suppose it is zero. Since the middle term is zero, the last term must also be zero. Using the second-order condition for a_0 , we see that in this case, $\lambda_2 = 0$. With $\lambda_2 = 0$, the first-order condition for S^0 at a_0 becomes

$$-U^P[X - S^0(X, t)] + \lambda_1 U_1^A[S^0(X, t), a_0] = 0 \quad \text{for all } X, t.$$

From this equation, we derive the fact that $0 \leq S_1^0 < 1$ for all X, t in the usual way. Furthermore, differentiating with respect to t , we obtain $S_2^0 \equiv 0$. But, the first term of (i) is (using integration by parts)

$$\begin{aligned} & \int_{a_0-\alpha_0}^{a_0+\alpha_1} \int_0^1 \{U^P[X - S^0(x, t)](1 - S_1^0) X_1 \\ & - U^P[X - S^0(x, t)] S_2^0\} f_\delta(t - a) d\theta dt. \end{aligned}$$

Therefore with $0 \leq S_1^0 < 1$ and $S_2^0 = 0$, the first term of (i) is positive, contradicting our initial assumption. Therefore, this term cannot be zero.

But, using integration by parts,

$$\begin{aligned}
 &W_3^P(0, 0, a_0) \\
 &= \int_{\alpha_0 - \alpha_0}^{\alpha_0 + \alpha_1} \int_0^1 U^P[X - S^0(X, t)](1 - S_1^0) X_1 - U^P[X - S^0(X, t)] S_2^0\} \\
 &\quad \times f_\delta(t - a) d\theta dt
 \end{aligned}$$

since $f_\delta(-\alpha_0) = F_\delta(-\alpha_0) = 0$. This shows that $W_3^P(0, 0, a_0) \neq 0$.

It remains to show that $(\partial a^*/\partial \epsilon)|_{\epsilon=\beta=0} \neq 0$. Using the second order condition for a and the fact that $a(0, 0) = a_0$, we need only show that $W_{13}^A(0, 0, a_0) \neq 0$. Now using $f_\delta(-\alpha_0) = h'(0) = 0$,

$$W_{13}^A(0, 0, a_0) = f'_\delta(-\alpha_0) \left\{ \int_0^1 U^A[S^0(X, a_0 - \alpha_0), a_0] d\theta - U^A(w, a_0) \right\}.$$

Since $f'_\delta(-\alpha_0) > 0$ and $w < \min_\theta S^0[X(a_0, \theta), a_0 - \alpha_0]$, we see that $W_{13}^A(0, 0, a_0) > 0$ and hence $(\partial a^*/\partial \epsilon)|_{\epsilon=\beta=0} > 0$. Thus $v_1^P(0, 0) \neq 0$. The remainder of the proof is identical to that of Proposition 7. Q.E.D.

Propositions 7 and 8 constitute the major positive implications of our model of imperfect monitoring. In order to generate further testable implications, we specialize our general assumptions. Further structure is imposed in stages and the implications derived at each stage.

Assumption 1. The production technology is given by $X(a, \theta) = a + \theta$ and θ is uniformly distributed on $[0, 1]$.⁹ Also, A 's utility function is separable, i.e., $U^A(w, a) = u(w) - g(a)$ with $g'' > 0$. Under this assumption, it can be shown that the optimal contract depending only on x is the optimal pure risk sharing arrangement (see Corollary 1).¹⁰ Thus, in this case, the optimal contract is identical to the one which would prevail if there were no moral hazard or incentive problems. For example, if the principal is risk neutral, the optimal contract specifies a constant payment to the agent and the agent's action will be the minimum possible.

This result can be extended to dichotomous contracts involving monitoring under the following additional

Assumption 2. The principal is risk neutral and the monitoring technology includes only monitors which are uniformly distributed on $[a - \epsilon, a + \epsilon]$ for some $\epsilon > 0$.

⁹ The results which follow also hold for the case $X(a, \theta) = a + g(\theta)$ where g is any strictly monotone (increasing or decreasing) function. This makes the uniformity assumption on θ somewhat less restrictive.

¹⁰ The proof uses the Euler-Lagrange equation and is similar to the proof of Corollary 1; it is omitted for the sake of brevity.

Under Assumption 2, the optimal monitoring contract is of the form

$$S^*(x, \alpha) = \begin{cases} w & \text{if } \alpha \geq a_1 - \epsilon \\ 0 & \text{otherwise,} \end{cases}$$

where w is a constant payment and $a_1 - \epsilon$ is the minimum acceptable value of α . Under this contract, the dismissal probability (this terminology is based on the interpretation of the agency relationship as an employment relation although other interpretations are possible) conditional on the action, a , is given by

$$p(a, a_1) = \begin{cases} 0 & \text{for } a \geq a_1 \\ (a_1 - a)/2\epsilon & \text{for } a_1 - 2\epsilon \leq a \leq a_1 \\ 1 & a \leq a_1 - 2\epsilon. \end{cases}$$

Thus, actions less than a_1 can be interpreted as shirking.

The actual values of the parameters w and a_1 will depend on the agent's utility function and his alternative opportunities. In order to derive more specific results, we make

Assumption 3. $u(w) = w^\gamma$, $g(a) = a^{1+\gamma}/(1 + \gamma)$ with $0 < \gamma < 1$ and the agent's expected utility is restricted to be nonnegative in solving the Pareto problem.

The second part of this assumption can be interpreted as assuming that the agent's only alternative is autarky, i.e., $w = a = 0$.

Using Assumptions 1-3, the optimal dichotomous monitoring contract is of the above form with values

$$w^* = (1 + \gamma) \epsilon, \tag{5}$$

$$a_1^* = 2\epsilon + \delta\gamma, \tag{6}$$

where $\delta = (2\epsilon^{1-\gamma})^{-1/\gamma}$. Moreover, in this case, the agent chooses action

$$a^* = \begin{cases} (1 + \gamma) \delta & \text{if } \epsilon \geq 2^{-(1+\gamma)} \\ a_1^* & \text{otherwise,} \end{cases} \tag{7}$$

and the resulting dismissal probability is given by

$$p^* = p(a^*, a_1^*) = \begin{cases} 1 - (2^{1+\gamma}\epsilon)^{-1/\gamma} & \text{if } \epsilon \geq 2^{-(1+\gamma)} \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

The expected utilities of the two parties for this contract are

$$E_{\alpha(a^*)} V^A(S^*, a^*) = 0, \tag{9}$$

$$E_{\alpha(a^*)} V^A(S^*, a^*) = \begin{cases} 1/2[1 + (1 + \gamma) \delta] & \text{if } \epsilon \geq 2^{-(1+\gamma)} \\ 1/2 + \delta\gamma + \epsilon(1 - \gamma) & \text{otherwise} \end{cases} > 1/2 \tag{10}$$

Since the expected utilities under the optimal nonmonitoring contract are 0 and $\frac{1}{2}$ respectively, this proves that dichotomous monitoring contracts are Pareto-superior, in this case, for any finite ϵ . From (5)–(8) and (10), it follows that when $\epsilon > 2^{-(1+\gamma)}$, w^* increases with ϵ , a_1^* decreases with ϵ , a^* decreases with ϵ , p^* increases with ϵ , and P 's utility decreases with ϵ . These results reinforce our intuition that as monitoring becomes less precise (i.e., as ϵ increases), the worker's wage contingent on not being dismissed must increase, the terms on which the worker is judged become more lenient, the worker invests less effort in this activity, the probability of being dismissed increases, and the gains to monitoring decrease to zero as the monitor becomes noisier. All these results are potentially testable. Although these results were derived for a special case, we conjecture that they carry over to more general contexts.

An interesting general result of this analysis is the following. Suppose the principal knew the agent's utility function and the production technology. He could then compute the agent's optimal action given any contract. In this case, there would be no doubt as to whether the agent were "shirking." Nevertheless, it would be impossible to motivate the agent to take a Pareto-superior action without the use of some sort of observable monitoring device. This could explain the observation that sometimes employers use costly monitoring devices even when all parties know that the employee is shirking in some sense.

6. SUMMARY AND CONCLUSIONS

In this paper we have investigated the relationship between the optimal incentive contract and the available information. We have shown that, except in the case that the agent is risk neutral, there is an incentive problem in the following sense. Given any contract and choice of action by an unsupervised agent, there is another contract and action which is Pareto-superior. Therefore, if supervision is feasible there are potential gains to such activity. We proceed to investigate how the results of supervising can optimally be incorporated in the contract. We also show that under certain conditions on the monitoring technology, it will always be optimal to incorporate the result of imperfect monitoring in a dichotomous contract. This contract stipulates that if the agent's action is found to be acceptable, then his share of the payoff depends on the monitoring result and the payoff. When the action is found to be unacceptable, the agent receives a constant which results in lower utility. Under special assumptions, we exhibit the Pareto-optimal monitoring contract. In this case, several interesting and testable implications are derived.

These results provide a rational explanation for many observed contractual relationships. For example, most employer–employee agreements stipulate

some remuneration to the employee provided that he is judged to have performed adequately. Otherwise the employee is discharged and therefore incurs considerable costs.

Another example is in the area of insurance contracts. Consider the owner of a new durable good, e.g., an automobile. Often the seller agrees to pay all or part of the cost of certain repairs provided that the buyer does not abuse the product. The determination of whether proper care is taken is based upon the regular maintenance record which is available to the seller during the guarantee period. Viewing the seller as the principal and the buyer as the agent, the treatment of the product as the agent's action and the maintenance record as the monitor, our results indicate that the warranty agreement will be a dichotomous contract, i.e., if the specified maintenance is performed, the seller pays some share of the cost of covered repairs (the share depending on the degrees of risk aversion of the buyer and seller and perhaps on the total cost of the repair), and if the maintenance is not performed, the buyer loses his warranty. This is, in fact, the standard warranty agreement in the case of automobiles where the maintenance record is a good estimator of actual treatment of the automobile. Several other applications are discussed in Harris and Raviv [4].

The situations dealt with in our model, although important, represent only a subset of the possible agency relationships. In particular, one aspect of the agency problem which is especially interesting and deserves further attention is the case in which the two individuals have different beliefs before entering into a relationship concerning the exogenous uncertainty. See Harris and Townsend [5, 6] for a discussion of this case.

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