Collusion and the incentives for information sharing

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Two steps are required for firms collusively to restrict output in stochastic markets. Firms must homogenize their market estimates by pooling information and they must cooperatively allocate production levels. In this article I examine the incentives for firms to share private information about a stochastic market. I show that there is never a mutual incentive for all firms in an industry to share unless they may cooperate on strategy once information has been shared. This situation is unfortunate, as society's welfare is maximized only when firms share information, but act competitively.

1. Introduction

■ Oligopolistic collusion in a stochastic market environment requires two steps. Firms must first agree on their estimates of the market state. Then they must cooperatively determine, and follow, an optimal strategy based on these homogenized beliefs. Most studies of cooperative oligopoly focus on the second step (Fellner, 1949; Telser, 1972; Friedman, 1977). A general conclusion is that given common information, collusion increases industry profits. In this article I concentrate on the first step—the incentives for firms to share private information about a stochastic market. I show that in a full Bayes-Cournot equilibrium, there is never a mutual incentive for all firms in an industry to share information. This situation is unfortunate as society's welfare is maximized only when firms share information, but act competitively. Thus, society faces a dilemma. Information pooling is good if firms behave competitively, but shared information makes anticompetitive agreements easier to construct.

The definition I use of a "full Bayes-Cournot" equilibrium assumes that firms make their quantity decisions based on their best Bayes estimates of their opponents' information. This equilibrium is formally identical to the "fulfilled expectations Bayes-Cournot" equilibrium specified in Novshek and Sonnenschein (1982). This study differs from Novshek and Sonnenschein's analysis in that I assume uncertain market variables may be parameterized by normal distributions. This allows the precise conditional expectations that characterize the equilibrium to be computed. Novshek and Sonnenschein make weaker distributional assumptions about the market's random variables. But to compute an equilibrium, they are forced to approximate firms' conditional expectations about other firms' data by assuming that each firm believes others' data are identical to their own. This simplifying assumption has several severe implications. It is only consistent with the rest of their model if firms' data noise variance is zero.[1] This forces the firms'

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[1] Formally stated, Vives (1982) shows that Novshek and Sonnenschein's (1982) assumption that $E(x|z_i) = E(z_j|z_i) = z_i$ for $i \neq j$, where x is the parameter to be estimated and z_i is firm i's signal, implies that the noise variance of z_i must be zero and thus $x \equiv z_i$, for all i. See the Appendix for more details.

optimal decision rules to be certainty-equivalent. Thus Novshek and Sonnenschein's principal result that firms are indifferent to complete information sharing, is valid only when firms are approximately certain of their environment. When conditional expectations are computed exactly, assuming normally distributed random variables, decision rules are not certainty-equivalent, and equilibria can be computed for a full range of data noise variances. Within this general framework it can be shown that all firms never wish to pool their information—except in the two special cases of zero or infinite data noise variance, when they are indifferent to pooling. In the sequel I call exactly computed equilibria, "full Bayes-Cournot" equilibria, and call equilibria computed by using Novshek and Sonnenschein's restriction on conditional expectations, "fulfilled expectations Bayes-Cournot" equilibria.

In Section 2, I discuss how market uncertainty affects incentives to collude. The influences of private and shared imperfect information on oligopolistic strategies along with an information structure are described in Section 3. A general Bayesian Cournot game model is presented and solved under imperfect information in Section 4. Section 5 attaches a simple oligopoly specification to the model of Section 4. The influences of varying accuracy of information on firm profits are displayed. In Section 6, I analyze the private incentives for firms to convert private into public information through pooling. The welfare effects of the various market outcomes are discussed in Section 7. Concluding remarks are in Section 8.

2. Uncertainty and collusion

■ Oligopolists' incentive to cooperate is very strong. By agreeing to restrict output, industry profits may be higher than if each firm acts independently. But stochastic market environments make successful collusion difficult. There are two reasons. First, as long as information (even if shared) is imperfect, firms are never sure of exact market conditions. When it is not possible to know the precise market state, Stigler (1964) points out that it is difficult to detect cheating on a collusive agreement, since perceived chiselling could be the result of outlying data observations rather than actual malicious behavior. In Green and Porter's (1981) dynamic oligopoly with trigger-price strategies, imperfect information similarly influences how frequently outputs will be collusively restricted or competitively expanded. A general conclusion of both models is that collusive output restriction is more moderate when imperfect information impedes the detection of cheaters. (See also Posner (1976) and Spence (1978).)

The second reason why imperfect market information may inhibit collusion is that if information is private (nonshared), firms may hold divergent views about market conditions. Lacking a confluence of opinion, firms find it difficult to agree on a cooperative strategy. An industry cartel would find it hard to determine optimal output shares if some members believe demand to be contracting by 5% while others believe it to be rising by 20%. For these two reasons, we should not expect collusive quantity setting in oligopolies where firms' information is imperfect and nonshared.

Information pooling facilitates collusion. It eliminates disagreements based on private information by allowing oligopolists to homogenize their perceptions of both the market state and other firms' information. Pooling also makes cheating more difficult and collusive quantity restriction more effective by improving the accuracy of every firm's market estimates. Information pooling may have a degrading effect on a firm's profits, though. When information is pooled, the quantity decisions of firms become more highly correlated. This may reduce the profits available to each firm. Trade associations are one example of information-sharing agreements. These groups collect private data from firms and disseminate it throughout the industry (Hay and Kelley, 1974; Fraas and Greer, 1977). Though information sharing should only arise if it raises firms' profits, it is also possible that sharing sufficiently mitigates risk so societal welfare is improved as well.[2]

In the sequel I show that if all firms voluntarily enter an information-pooling agreement, it means they expect to set outputs in an anticompetitive fashion—with lower social welfare than if the firms set competitive quantities solely on the basis of their private information.

3. Information structure

■ The stochastic oligopoly considered here is a Bayesian game as formalized by Harsanyi (1967, 1968a, 1968b). Firms know their own profit functions and the profit functions of their rivals, but are imperfectly informed about variables such as cost or demand that specify the market environment or state. This uncertainty may impinge on the firm in two ways. First, since the exact environment is unknown, firms must collect information and form estimates of the market state. Second, since the noisy information available to one firm may not be shared by other firms, firms must estimate the state estimates held by other firms. While the first influence of uncertainty arises in any market, the latter only occurs in oligopolistic markets where all information is not public. If all information is pooled, firms know precisely the state estimates held by other firms. And if markets are not oligopolistic with recognized mutual interdependence, the influence of nonshared information is irrelevant.[3]

Before introducing the particular information structure studied, I describe the other game elements. State variables are the elements such as demand and costs characterizing the market environment. Control variables are instruments firms use to maximize profits—where profits are a function of both the state and control variables. Strategies are decision rules firms use to select values for their control variables. Firms' decisions must be conditioned on their available information. The following notation is used:

x = vector representing the market state; $x \in X$ set of all possible state values; there may be a probability distribution given over X.

y_i = information vector available to firm i; $y_i \in Y_i$ set of all possible information values for firm i, $i = 1, \ldots, N$.

$u_i = \gamma_i(y_i)$: control value chosen by firm i, $u_i \in U_i$ set of all possible control values for firm i; $\gamma_i(\cdot)$ is firm i's strategy or decision rule, $\gamma_i \in \Gamma_i$ the set of all Borel measurable functions mapping the information space Y_i into the decision space U_i.

The information structure of the game is a description of the range of each firm's knowledge about the state, and about other firms' information. Individual firm information sets include noisy data measurements of the state and *a priori* beliefs. The noisy state measurements are represented by the random vector:

z_i = measurement of state x received by firm i, $z_i \in Z_i$ set of all possible measurements.

These data are related to the true state value x via the observation equation:

$$z_i = x + v_i, \tag{1}$$

[2] Posner (1976) also makes this point, i.e., information exchange aids collusion, but in the absence of collusion, such exchange is socially desirable.

[3] This characterization of noisy information as shared or private corresponds to Harsanyi's (1967, 1968a, 1968b) classification of imperfect versus incomplete information, i.e., firms are uncertain of exogenously given market states (imperfect information), but may also be ignorant of rival firms' information (incomplete information).

where v_i is Gaussian white noise with mean zero and covariance matrix R_i, $v_i \sim N(0, R_i)$. Measurement z_i conveys perfect information if the distribution of v_i is degenerate, tr $\{R_i\} = 0$. Measurements are imperfect or noisy if v_i has a nondegenerate distribution, tr $\{R_i\} > 0$. For convenience, I assume that the measurements received by each firm are independent of one another. But even if they were correlated, no qualitative results in this article would be altered.

A portmanteau variable $\theta \in \Theta$ contains the *a priori* information. Lack of a subscript indicates that all firms share the same *a priori* information. If this were not true, there would be severe difficulties, dealt with in Aumann (1976), Harsanyi (1967, 1968a, 1968b), and Geanakoplos and Polemarchakis (1982), concerning the specification of common knowledge. This common *a priori* information is assumed to contain the accuracy of each firm's data measurements, $R_1, \ldots, R_N$, and the prior probability distribution over x. This prior on x is also Gaussian, $x \sim N(\mu, M)$. Thus θ represents the collection, $\theta = \{\mu, M, R_1, \ldots, R_N\}$. Firm i's complete information vector may now be denoted:

$$y_i = \{z_i, \theta\}, \qquad i = 1, \ldots, N. \tag{2}$$

Firms may pool their private information z_i by constructing and sharing the sufficient statistic $z = R \sum_{i=1}^{N} R_i^{-1} z_i$. This statistic has a Gaussian distribution, $z \sim N(x, R)$, where $R = [\sum_{i=1}^{N} R_i^{-1}]^{-1}$. Hence, under shared information all firms have the identical information vectors:

$$y = \{z, \theta\}, \qquad \theta = \{\mu, M, R\}. \tag{3}$$

In the following section, I define the game equilibrium and demonstrate the firms' optimal strategies under these two information structures.

4. Equilibrium

■ The game model is N-player, nonzero-sum. Noncooperative behavior is assumed to result in a Cournot equilibrium. In such an equilibrium no firm may improve its payoff by any unilateral action. Cooperative behavior is assumed to result in the monopoly solution.[4]

A linear-quadratic payoff structure is necessary to show the existence of a unique, noncooperative Cournot equilibrium. While this restriction is severe, the linear-quadratic may be considered a second-order approximation to more general functional forms. This structure also accords with related models in the literature by Radner (1962) and Basar and Ho (1974).

The profit function for firm i may be written:

$$\pi_i(x, u_1, \ldots, u_N) = u_i'C_i x + \tfrac{1}{2} u_i' D_{ii} u_i + \sum_{j \neq i} u_i' D_{ij} u_j, \tag{4}$$

where C_i, D_{ii}, and D_{ij} are conformable parameter matrices with D_{ii} negative definite. Expected or average profits for firm i are given by:

$$J_i(\gamma_1, \ldots, \gamma_N) = E[\pi_i(x, u_1, \ldots, u_N) | u_j = \gamma_j(\cdot),\ j = 1, \ldots, N], \tag{5}$$

where expectation is taken over the prior distributions of the random variables x, and $v_1, \ldots, v_N$.

[4] This is, of course, a naive view of cooperative oligopoly. I use it only as a convenient point of comparison with noncooperative oligopoly. For a more sophisticated look at cooperative oligopoly, see Telser (1972) or Friedman (1977).

Definition: A set of strategies $\{\gamma_1^*, \ldots, \gamma_N^*\}$ define a full Bayes-Cournot equilibrium if:

$$J_i(\gamma_1^*, \ldots, \gamma_N^*) \geq J_i(\gamma_1^*, \ldots, \gamma_{i-1}^*, \gamma_i, \gamma_{i+1}^*, \ldots, \gamma_N^*) \quad \text{for all} \quad \gamma_i \in \Gamma_i, \quad i = 1, \ldots, N.$$

Theorem 3 of Basar (1978) demonstrates that under certain conditions on the D_{ii} and D_{ij} matrices—which are satisfied in any well-posed oligopoly specification—a unique set of equilibrium strategies $\{\gamma_1^*, \ldots, \gamma_N^*\}$ exists. Theorem 4 of Basar (1978) shows these decision rules are linear with the form:

$$\gamma_i^*(y_i) = A_i\mu + B_i(d_i - \mu), \qquad i = 1, \ldots, N, \tag{6}$$

where

$$d_i = E[x|y_i] = \mu + G_i(z_i - \mu), \qquad i = 1, \ldots, N, \tag{7}$$

is firm i's Bayes posterior estimate for the state x; and A_i and B_i, $i = 1, \ldots, N$, are the unique solutions to:

$$D_{ii}A_i + \sum_{j \neq i} D_{ij}A_j = -C_i, \qquad i = 1, \ldots, N, \tag{8a}$$

$$D_{ii}B_i + \sum_{j \neq i} D_{ij}B_jG_j = -C_i, \qquad i = 1, \ldots, N. \tag{8b}$$

The matrix G_i is called the filter gain matrix. It is computed from the covariances of the prior and data. Since prior and data are both assumed normal, a straightforward application of Bayes' rule shows that

$$G_i = (M^{-1} + R_i^{-1})^{-1}R_i^{-1}.\text{[5]} \tag{9}$$

If firms pool information, a special case of the above reasoning shows that each firm's strategy is:

$$\gamma_i^*(y) = A_id, \qquad i = 1, \ldots, N, \tag{10}$$

where A_i is derived from (8a), and the common posterior Bayes state estimate d is computed:

$$d = E[x|y] = \mu + G(z - \mu), \tag{11}$$

$$G = (M^{-1} + R^{-1})^{-1}R^{-1}, \tag{12}$$

and z and R are given as in Section 3.

In the Appendix, I demonstrate on a heuristic level the above derivations. For a rigorous proof, the reader is referred to Basar (1978).

Before presenting the actual oligopoly specification, I shall make a few observations about these results. First, note that the solution to the shared information game is certainty-equivalent, i.e., if d were known to estimate the state exactly, $d = x$, then the firm's optimal decision would still be $u_i^* = A_id = A_ix$. When there is both private and shared information, the firm's decision rule is not certainty-equivalent. This is evident from (8b), as the B_i coefficients are functions of the filter gain matrices G_j. Thus, data noise influences the optimal decision rule. If a fulfilled expectations equilibrium was computed in the manner of Novshek and Sonnenschein (1982), then $B_i = A_i$, independent of the G_j (see the Appendix for details). In such an equilibrium, the firm's decision rule would reduce to $\gamma_i^{FE}(y_i) = A_id_i$. The difference between this certainty-equivalent action and the optimal action under full Bayes equilibrium is $\gamma_i^{FE}(y_i) - \gamma_i^*(y_i) = (A_i - B_i)(d_i - \mu)$, where B_i is computed as in (8b). This term, which Novshek and Sonnenschein neglect, is always nonzero unless the firm's data are identical to its prior ($z_i = \mu = d_i$), which only arises if data noise variances r_i all equal zero.

[5] See DeGroot (1970) for a fuller discussion.

5. Oligopoly specification

■ Within the linear-quadratic framework, I specify a particularly simple, but popular, oligopoly. For clarity, I initially assume two firms. Later in this section, I display the oligopoly equilibrium for N firms.

Firms sell a homogeneous good with constant unit costs of production. They face a linear demand curve with a random intercept. Thus, market price net of unit costs is

$$p = a - b \sum_{j=1}^{N} u_j, \tag{13}$$

where a is the random demand intercept minus unit costs, b is a known slope parameter, and u_j is the output of firm j. Firms choose outputs u_i (controls) to maximize profits:

$$\pi_i = pu_i. \tag{14}$$

This oligopoly corresponds to the general model in Section 4 if we associate the random intercept a with the state variable x; u_j, $j = 1, \ldots, N$, with the control variables; and $C_i = 1$; $D_{ii} = -2b$; $D_{ij} = -b$, $i \neq j$.

Since the state vector is a scalar, each firm's posterior state estimate is $d_i = \mu + g_i(z_i - \mu)$, where $g_i = m/(m + r_i)$. Lower-case letters correspond to the scalar value of the matrix denoted by the upper-case letter. The decision rule coefficients for the $N = 2$ case are:

$$A_i = \frac{1}{3b}, \qquad i = 1, 2, \tag{15}$$

$$B_1 = \frac{2 - g_2}{b(4 - g_1 g_2)}, \qquad B_2 = \frac{2 - g_1}{b(4 - g_1 g_2)}. \tag{16}$$

Pooled information results in the posterior state estimate $d = \mu + g(z - \mu)$, where $g = m/(m + r)$ and $r = (r_1 r_2)/(r_1 + r_2)$. If we rewrite g in terms of g_1 and g_2, we have:

$$g = \frac{g_1 + g_2 - 2g_1 g_2}{1 - g_1 g_2}. \tag{17}$$

Under pooled information and competitive behavior, the control law coefficients remain as in (15). If firms use pooled information to produce as cooperative joint profit maximizers, it is simple to show that optimal industry output is $u^* = d/(2b)$.

The quality of a firm's information is related to its filter gain g_i. For any fixed level of prior covariance m, g_i increases from 0 to 1 as r_i declines from ∞ to 0. If firm i's data are very inexact ($r_i \to \infty$), then its filter gain approaches zero ($g_i \to 0$). As firm i's data accuracy improves ($r_i \to 0$), its filter gain approaches one ($g_i \to 1$). In the sequel, I assume that m is fixed. Hence, there is a monotonic relation between the magnitude of g_i and the accuracy of firm i's data. I also set $\mu = 0$. This simplifies notation without any loss of generality.

Expected Cournot profits for firm i under private information may be computed by substituting optimal controls, $u_1^* = B_1 d_1$, $u_2^* = B_2 d_2$, into (14):

$$J_i(\gamma_1^*, \gamma_2^*) = E[(a - b(B_1 g_1 z_1 + B_2 g_2 z_2))B_i g_i z_i]. \tag{18}$$

Denoting this expression by π_i^p, and remembering that $E[az_i] = m$, $E[z_i^2] = m + r_i$, and $E[z_i z_j] = m$ for $i \neq j$, with some straightforward calculation we reduce this to:

$$\pi_i^p = bB_i^2 g_i m, \qquad i = 1, 2. \tag{19}$$

Under information sharing expected profits for each firm are:

$$\pi_i^s = \frac{gm}{9b}, \qquad i = 1, 2. \tag{20}$$

If firms act cooperatively, it is not possible to determine precisely the profits earned by each firm, since the division of industry profits among firms may be colored by differences in relative bargaining power. However, expected industry profits under cooperative quantity setting would be:

$$\pi^c = \frac{gm}{4b}. \tag{21}$$

For an N-firm oligopoly, the optimal strategy coefficients are $A_i = 1/(N + 1)b$ for $i = 1, \ldots, N$. The B_i coefficients are computed from (8b). Firm i's profits in a private information equilibrium are then given by (19). If all N firms pool their information, each firm's common filter gain would be:

$$g = \frac{\sum_{i=1}^{N} [g_i \prod_{j \neq i} (1 - g_j)]}{\sum_{i=1}^{N} [g_i \prod_{j \neq i} (1 - g_j)] + \prod_{i=1}^{N} (1 - g_i)} \tag{22}$$

Under competitive play, this shared information yields each firm a profit of:

$$\pi_i^s = \frac{gm}{(N+1)^2 b}, \qquad i = 1, \ldots, N. \tag{23}$$

If shared information results in cooperative play, industry profits are the same as (21).

Before comparing profit levels under the different information structures, it is useful to verify the effects of information accuracy on firm profits. Differentiating π_i^s and π^c with respect to g shows that raising the accuracy of group information increases profits in either the shared noncooperative or shared cooperative oligopolies. Since $\partial g/\partial g_i > 0$ for all i, raising the accuracy of one firm's data also has the same effect if data are shared.

If, however, firms are restricted to their private information, an increase in the accuracy of one firm's data raises its expected profit, but lowers its opponents' expected profit. For the two-firm case this is easily demonstrated:

$$\frac{\partial \pi_1^p}{\partial g_1} = 2bB_1 \frac{\partial B_1}{\partial g_1} g_1 m + bB_1^2 m > 0$$

because

$$\frac{\partial B_1}{\partial g_1} = \frac{g_2(2 - g_2)}{b(4 - g_1 g_2)^2} > 0 \quad \text{and} \quad B_1 > 0 \quad \text{for} \quad 0 \le g_1, g_2 \le 1;\ b > 0;$$

and

$$\frac{\partial \pi_2^p}{\partial g_1} = 2bB_2 \frac{\partial B_2}{\partial g_1} g_2 m < 0$$

because

$$\frac{\partial B_2}{\partial g_1} = \frac{-2(2 - g_2)}{b(4 - g_1 g_2)^2} < 0 \quad \text{and} \quad B_2 > 0 \quad \text{for} \quad 0 \le g_1, g_2 \le 1;\ b > 0.$$

An inductive argument establishes these properties for N-firm oligopolies.

Whether improvements in one firm's information raise overall industry profits is ambiguous. Examine the two-firm case. Differentiating expected industry profits with respect to g_1 gives:

$$\frac{\partial(\pi_1^p + \pi_2^p)}{\partial g_1} = \frac{bmB_1^2}{(2 - g_2)(4 - g_1 g_2)^2} [8 + 6g_1 g_2 - g_1 g_2^2 - 12 g_2].$$

The sign of this expression is the same as the sign of the expression in brackets. It is generally positive except for large values of g_2 coupled with small values of g_1. In this case it is negative. Thus, in a two-firm oligopoly improved accuracy for one firm generally raises industry profits, unless that firm is quite ignorant, while its rival is quite knowledgeable.

6. Incentives for firms to share

■ Are there private incentives for firms in an industry to trade their private information for pooled information of higher accuracy? Given the positive influence of own information accuracy on own profits and its negative influence on other's profits, it would appear that firms with less accurate information than their rivals' would wish to share, but firms with sufficiently more accurate information would not wish to pool. Are there combinations of information accuracies, say when firms have about equal data precision, that yield each firm a higher profit under sharing than under using privately obtained information? The answer is no. If firms have equally accurate information, they cannot improve their profits by sharing. A firm may only improve profits by sharing if its information is substantially less accurate than its competitors'—but in this event, its competitors will not wish to give up their information advantage through sharing. This is easiest to demonstrate for the two-firm case. We seek values of g_1, g_2, and g such that $\pi_1^p \gtreqless \pi_1^s$ for $0 \leq g_1, g_2 \leq 1$:

$$\pi_1^p = bmB_{1}^2 g_1 \gtreqless \frac{gm}{9b} = \pi_1^s$$

$$\frac{m}{b}\frac{4g_1 - 4g_1g_2 + g_1g_2^2}{16 - 8g_1g_2 + g_1^2g_2^2} \gtreqless \frac{m}{b}\frac{g_1 + g_2 - 2g_1g_2}{9 - 9g_1g_2} \tag{24}$$

or

$$2g_1^3g_2^3 - 10g_1^2g_2^3 + 20g_1^2g_2^2 - g_1^3g_2^2 - 28g_1^2g_2 + 17g_1g_2^2 - 4g_1g_2 + 20g_1 - 16g_2 \gtreqless 0.$$

There are no roots to this expression for $0 < g_2 \leq g_1 < 1$. It is always positive in this region. Only when $g_1 = g_2 = 0$ or $g_1 = g_2 = 1$ are firms just indifferent between sharing and keeping information private (Clarke, 1983). This latter case of zero noise variance displays the indifference result of Novshek and Sonnenschein (1982). Otherwise there are no (g_1, g_2) combinations where there is a joint incentive to share information. Equations comparing π_i^p with π_i^s in N-firm oligopolies are more elaborate than the two-firm equation (24). But straightforward algebraic induction reveals that the qualitative results remain the same: all firms cannot gain by sharing information.

This indicates that under the requirement of noncooperative behavior, we should not see all firms in an industry freely sharing information. It may still be that firms are willing to sell information. This possibility would exist if total industry profits (producer surplus) under shared information exceed the producer surplus under private information. Except in one case, however, there are insufficient profit increments accruing to the less knowledgeable firms under information sharing to enable them to compensate adequately the more knowledgeable firms for their profit losses due to sharing. This special case occurs in a two-firm oligopoly. Compute the (g_1, g_2) combinations such that $\pi_1^p + \pi_2^p \gtreqless \pi_1^s + \pi_2^s$ for $0 \leq g_1, g_2 \leq 1$:

$$\pi_1^p + \pi_2^p \gtreqless \pi_1^s + \pi_2^s$$

$$\frac{m}{b}\frac{4g_1 + 4g_2 - 8g_1g_2 + g_1^2g_2 + g_1g_2^2}{16 - 8g_1g_2 + g_1^2g_2^2} \gtreqless \frac{m}{b}\frac{2g_1 + 2g_2 - 4g_1g_2}{9 - 9g_1g_2}. \tag{25}$$

For nearly all values of (g_1, g_2), private information profit-dominates shared information. Only if one firm has extremely accurate information, $g_i > .96$, while the other firm's information is also not unduly inaccurate, $g_j > .57$, are industry profits improved by information sharing. Hence, the firm with bad information may be able to bribe the firm with good information to share its knowledge. For oligopolies of three or more firms, there are no filter gain combinations where producer surplus rises from universal information pooling.

So far I have assumed that information sharing does not lead to cooperative play. If cooperative quantity setting is possible, then there is always an incentive to share—as long as a suitable profit distribution can be negotiated among the conspirators. To see this, compare industry profits under private information, $\sum_{i=1}^{N} \pi_i^p$, with industry profits under shared information with cooperative play from (21). Calculations verify that $\sum_{i=1}^{N} \pi_i^p \leq \pi^c$ for all $(g_1, \ldots, g_N)$ with equality only when all but one g_i equals zero. This results because, for example, if $g_j > 0$, $g_i = 0$ for all $i \neq j$, then $g = g_j$ and $\sum_{i=1}^{N} \pi_i^p = \pi_j^p = \pi^c$. But if information sharing with cooperative play is associated with an equal distribution of profits, i.e., $\pi_1^c = \ldots = \pi_N^c = \pi^c/N$, then firms with sufficiently better than average information may not wish to join the cartel. Hence, cartels would only form in industries where firms have approximately equal knowledge.

In sum, we have seen that if firms must act noncooperatively, they will never all agree to share information gratuitously. If firms may make payments to other firms in return for revealing private information, there are a few duopoly (g_1, g_2) combinations where industry profits increase so that a compensation scheme is feasible. If cooperative behavior is permitted, then industry profits will unambiguously improve if all firms share their information and act jointly. While there is some leeway in determining the share of profits to be parcelled out to each firm under such an arrangement, it is limited by the security level that each firm may attain by keeping its information private. Thus profit shares under cooperation must follow information shares fairly closely.

7. Welfare

■ In the previous section, I have discussed the effects of information accuracy and sharing on producer surplus under noncooperative and cooperative behavior. In this section, I examine the effects of these factors on consumer surplus and overall welfare. Under private information, the expected value of producer surplus may be computed from (19) as

$$PS^p = \sum_{i=1}^{N} \pi_i^p. \tag{26}$$

Expected consumer surplus is

$$CS^p = \tfrac{1}{2}\left(m \sum_{i=1}^{N} B_i g_i - PS^p\right), \tag{27}$$

and expected total surplus (welfare) is

$$W^p = PS^p + CS^p. \tag{28}$$

If all firms share information and choose competitive quantities, expected producer surplus is computed from (23),

$$PS^s = \sum_{i=1}^{N} \pi_i^s = \frac{Ngm}{(N+1)^2 b}, \tag{29}$$

expected consumer surplus is

$$CS^s = \frac{N^2mg}{2(N+1)^2b}, \tag{30}$$

and expected welfare is

$$W^s = PS^s + CS^s. \tag{31}$$

When all firms share information and cooperatively select quantities, expected producer surplus comes from (21),

$$PS^c = \frac{gm}{4b}, \tag{32}$$

expected consumer surplus is

$$CS^c = \frac{gm}{8b}, \tag{33}$$

and expected welfare is

$$W^c = PS^c + CS^c. \tag{34}$$

It is straightforward to verify that each total surplus rises when any player's information improves. Hence information is always a "good" for society—assuming a fixed regime of noncooperation or cooperation. The important issue, though, is how consumer surplus and welfare changes if private information becomes shared, or play switches from competitive to cooperative. The following set of inequalities describes the relative surplus levels under the different regimes:

$$PS^c \geq PS^p \gtreqless PS^s,^{6}$$

$$CS^s \geq CS^p \geq CS^c,$$

$$W^s \geq W^p \geq W^c.$$

Thus society and consumers' first preference is to have information shared among firms choosing competitive quantities. But if cooperative behavior cannot be prevented once information is shared, a second-best equilibrium might be to ban universal information transfers.

8. Concluding remarks

■ This article has demonstrated that in a general "full Bayes-Cournot" equilibrium, universal information sharing will not take place in a competitive world (except perhaps when firms are perfectly informed or completely ignorant and they are indifferent to sharing). This result simplifies policy issues. If all industry firms are observed to pool information without paying each other compensation, they must be setting quantities cooperatively on the basis of the homogenized information.[7] Hence information-pooling mechanisms like trade associations can be considered *prima facie* evidence that firms are illegally cooperating to restrict output. This result strengthens Posner's (1976) informal analysis of the desirability of information-sharing agreements. On the other hand, lack of information-pooling mechanisms can be taken as fairly good evidence that cooperative behavior is impeded.

Given that firms have no individual incentive to share information in the absence of collusive quantity setting, antitrust authorities are left with a difficult choice. They can

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[6] The lack of a strict ordering between PS^p and PS^s occurs only in duopolies, and was discussed in Section 5. For $N > 2$ firms, $PS^p \geq PS^s$.

[7] Even if compensation is paid, there is only an extremely limited range of information combinations (g_1, g_2) where noncooperative behavior under shared information allows higher industry profits than private information.

attempt to promote or subsidize information transfer while making redoubled efforts to suppress cooperative play. Or, if they think that shared information presents just too inviting an environment for cooperative behavior, and they doubt their ability to control cooperation in an environment of homogeneous information, they may seek a safer, second-best equilibrium by simply prohibiting information transfer.

In the foregoing I have assumed that the cooperative outcome is equivalent to the monopoly outcome. If such tight collusion is not achievable, firms are even less likely to pool information voluntarily.

This investigation has demonstrated that under a very general equilibrium concept, strong profitability and welfare conclusions can be drawn about information sharing. While the stochastic game model used in this article is quite simple, it is also quite general, and is a valuable tool for modeling many facets of oligopoly behavior under imperfect information.

Appendix

■ This appendix presents a heuristic, two-firm sketch of how the game model may be solved for its Nash equilibrium. The complete proof is in Basar (1978).

First-order conditions on (5) are:

$$u_i = -D_{ii}^{-1}C_iE[x|y_i] - D_{ii}^{-1}D_{ij}E[u_j|y_i], \qquad i = 1, 2. \tag{A1}$$

Since $u_i = \gamma_i(y_i)$, $i = 1, 2$, we have:

$$\gamma_i(y_i) = -D_{ii}^{-1}C_iE[x|y_i] - D_{ii}^{-1}D_{ij}E[\gamma_j(y_j)|y_i], \qquad i = 1, 2. \tag{A2}$$

Now substituting in for $\gamma_2(y_2)$, we get

$$\gamma_1(y_1) = -D_{11}^{-1}C_1E[x|y_1] - D_{11}^{-1}D_{12}E[-D_{22}^{-1}C_2E[x|y_2] - D_{22}^{-1}D_{21}E[\gamma_1(y_1)|y_2]|y_1], \tag{A3}$$

which is a functional equation in $\gamma_1(y_1)$. This may be rewritten as

$$\gamma_1(y_1) = h_1(y_1) + H_1(\gamma_1(y_1)), \tag{A4}$$

where

$$h_1(y_1) = -D_{11}^{-1}C_1E[x|y_1] + D_{11}^{-1}D_{12}D_{22}^{-1}C_2E[E[x|y_2]|y_1] \tag{A5}$$

$$H_1(\cdot) = D_{11}^{-1}D_{12}D_{22}^{-1}D_{21}E[E[\cdot|y_2]|y_1]. \tag{A6}$$

Since it may be shown that the operator $E[\gamma_j(y_j)|y_i]$ is a contraction mapping, a sufficient condition to ensure a unique fixed function in the Banach space generated by the linear operators $E[x|y_i]$ and $E[\gamma_j(y_j)|y_i]$ is that

$$\lambda|D_{11}^{-1}D_{12}D_{22}^{-1}D_{21}| < 1, \tag{A7}$$

where $\lambda|D|$ represents the maximum eigenvalue of $D'D$.

The control law (6) follows by construction. To simplify matters, assume $\mu = 0$. Now:

$$E[x|y_i] = d_i = G_iz_i, \tag{A8}$$

$$E[E[x|y_j]|y_i] = E[G_jz_j|y_i] = G_jE[z_j|y_i] = G_jG_iz_i. \tag{A9}$$

Since $\gamma_i(y_i) = B_id_i = B_iG_iz_i$, we have

$$E[E[\gamma_i(y_i)|y_j]|y_i] = B_iG_iG_jG_iz_i. \tag{A10}$$

Substituting (A8), (A9), and (A10) into (A3) gives:

$$B_1 = -D_{11}^{-1}C_1 + D_{11}^{-1}D_{12}D_{22}^{-1}C_2G_2 + D_{11}^{-1}D_{12}D_{22}^{-1}D_{21}B_1G_1G_2, \tag{A11}$$

which is the equation for B_1 given in (8b). Substituting (A11) into (A2) will give B_2.

The case of shared information is just a special case of the foregoing. If $y_1 = y_2$, then (A9) and (A10) become:

$$E[E[x|y_j]|y_i] = Gz \tag{A12}$$

$$E[E[\gamma_i(y_i)|y_j]|y_i] = B_iGz. \tag{A13}$$

Substituting into (A3) gives

$$B_1 = -D_{11}^{-1}C_1 + D_{11}^{-1}D_{12}D_{22}^{-1}C_2 + D_{11}^{-1}D_{12}D_{22}^{-1}D_{21}B_1, \tag{A14}$$

which is identical to A_1 given in (8a).

The Novshek and Sonnenschein (1982) method of computing their fulfilled expectations equilibrium would alter (A9) and (A10) by forcing $E[E[x|y_j]|y_i] = G_iz_i$ and $E[E[\gamma_i(y_i)|y_j]|y_i] = B_iG_iz_i$. This results in $B_i = A_i$, thus the certainty-equivalent decision rule.

References

AUMANN, R. "Agreeing to Disagree." *Annals of Statistics*, Vol. 4 (1976), pp. 1236–1239.

BASAR, T. "Decentralized Multicriteria Optimization of Linear Stochastic Systems." *IEEE Transactions on Automatic Control*, Vol. 23 (April 1978), pp. 233–243.

——— AND HO, Y.C. "Informational Properties of the Nash Solutions of Two Stochastic Nonzero-Sum Games." *Journal of Economic Theory*, Vol. 7 (1974), pp. 370–387.

CLARKE, R.N. "Duopolists Don't Wish to Share Information." *Economics Letters*, Vol. 11 (1983), pp. 33–36.

DEGROOT, M. *Optimal Statistical Decisions*. New York: McGraw-Hill, 1970.

FELLNER, W. *Competition among the Few*. New York: Knopf, 1949.

FRAAS, A. AND GREER, D. "Market Structure and Price Collusion: An Empirical Analysis." *Journal of Industrial Economics*, Vol. 26 (September 1977), pp. 29–33.

FRIEDMAN, J. *Oligopoly and the Theory of Games*. Amsterdam: North-Holland, 1977.

GEANAKOPLOS, J. AND POLEMARCHAKIS, H. "We Can't Disagree Forever." *Journal of Economic Theory*, Vol. 28 (October 1982), pp. 192–200.

GREEN, E. AND PORTER, R. "Noncooperative Collusion under Imperfect Price Information." Center for Economic Research Discussion Paper No. 81-142, University of Minnesota, February 1981.

HARSANYI, J. "Games with Incomplete Information Played by Bayesian Players: Parts I, II and III." *Management Science*, Vol. 14 (November 1967), pp. 159–182, Vol. 15 (January 1968a), pp. 320–334, Vol. 15 (March 1968b), pp. 486–502.

HAY, G. AND KELLEY, D. "An Empirical Survey of Price-Fixing Conspiracies." *Journal of Law and Economics*, Vol. 17 (April 1974), pp. 13–38.

NOVSHEK, W. AND SONNENSCHEIN, H. "Fulfilled Expectations Cournot Duopoly with Information Acquisition and Release." *Bell Journal of Economics*, Vol. 13 (Spring 1982), pp. 214–218.

POSNER, R. *Antitrust Law: An Economic Perspective*. Chicago: University of Chicago Press, 1976.

RADNER, R. "Team Decision Problems." *Annals of Mathematical Statistics*, Vol. 33 (1962), pp. 857–881.

SPENCE, A.M., "Tacit Coordination and Imperfect Information." *Canadian Journal of Economics*, Vol. 11 (August 1978), pp. 490–505.

STIGLER, G.J. "A Theory of Oligopoly." *Journal of Political Economy*, Vol. 72 (February 1964), pp. 44–61.

TELSER, L. *Competition, Collusion, and Game Theory*. Chicago: Aldine-Atherton, 1972.

VIVES, X. "Duopoly Information Equilibrium: Cournot and Bertrand." Department of Economics, University of California-Berkeley, September 1982.