Econometrica, Vol. 54, No. 4 (July, 1986), 923-942

COMMON AGENCY

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We extend the principal-agent framework with risk-neutral principals to situations in which several principals simultaneously and independently attempt to influence a common agent. We show that implementation is, in the aggregate, always efficient (cost-minimizing), and that noncooperative behavior induces an efficient (potentially second-best) action choice if and only if collusion among the principals would implement the first-best action at the first-best level of cost. We also investigate the existence of equilibria, the distribution of net rewards among principals, the characteristics of actions chosen in inefficient equilibria, and potential institutional remedies for welfare losses induced by noncooperative behavior.

KEYWORDS: Principal-agent, several principals, efficiency, remedies.

1. INTRODUCTION

THE BILATERAL PRINCIPAL-AGENT MODEL (c.f., Holmstrom (1979), Grossman and Hart (1983)) provides a highly flexible framework for studying a variety of important economic phenomena, ranging from planning problems (e.g., the design of optimal social insurance programs, as in Diamond and Mirrlees (1978)) to contractual relations (e.g., the optimal structure of sharecropping arrangements, as in Stiglitz (1974)).[2] Frequently, however, the action chosen by a particular individual (the agent) affects not just one, but *several* other parties (the principals), whose preferences for the various possible actions typically conflict. We refer to such situations as instances of "common agency." As with bilateral agency, examples of common agency can be seen as falling into one of two categories, according to whether agency is *delegated* or *intrinsic.*

Delegated common agency arises when several parties voluntarily (and perhaps independently) bestow the right to make certain decisions upon a single (common) agent. Such arrangements are particularly prevalent in wholesale trade. Numerous products are marketed through merchandise agents and brokers (such as commission merchants and manufacturers' agents), who often represent the potentially conflicting interests of several principals.[3] In fact, the 1972 Census of Wholesale Trade revealed that, of $695 billion in wholesale trade, over $85 billion was transacted through merchandise agents and brokers ($19 billion through

[1] We are particularly grateful to an anonymous referee whose insightful comments led both to a strengthening of our results and to substantial expositional simplification. We would also like to thank Franklin Fisher, Eric Maskin, and seminar participants at Harvard, Stanford, U.C.S.D., University of Chicago, and Yale for helpful comments. The second author gratefully acknowledges financial support from the Sloan Foundation through its grant to the MIT Department of Economics.

[2] Other work on the bilateral agency model includes important papers by Arrow (1971), Ross (1973), and Mirrlees (1975, 1979).

[3] While many manufacturers' agents refuse to serve two principals with directly competing products, product lines may be quite similar. Even where substitutability between principals' products is low, conflict still arises over the allocation of the agent's marketing efforts.

commission merchants, and $23 billion through manufacturers' agents in particular).[4] Similar institutions, though not as prevalent, are also observed in a number of retail industries, including travel, insurance, and real estate.[5] *Intrinsic* common agency, on the other hand, arises when an individual is "naturally" endowed with the right to make a particular decision affecting other parties, who may in turn attempt to influence that decision. Planning problems fall into this category, since the actions of each citizen impinge upon the planners' objectives, and since planners are "constitutionally" endowed with the power to impose incentive schemes (tax or regulate). More often than not, a particular individual or firm is subject to the strictures of not one, but several planners (levels of government, disparate agencies within the same level, or superiors), each of which pursues different objectives. For example, although both state and federal governments impose income taxes (corporate and personal), they may prefer quite different balances between equity and efficiency. Similarly, government agencies are frequently accused of pursuing narrow legislative mandates, to the detriment of social objectives which fall under the auspices of other regulatory bodies. Intrinsic common agency appears in nongovermental planning problems as well. Within firms possessing matrix organizations, subordinates (e.g., brand managers) are responsible to several superiors (e.g., both the marketing and financial vice-presidents), each of whom exercises some control over financial reward.[6]

In all the cases mentioned above, the principals could potentially benefit from mutual cooperation. When principals act collectively, application of the bilateral agency framework (treating the set of principals as a single entity) is appropriate. Thus, the relationship between the stockholders and managers of a publicly held corporation may be treated as a traditional principal-agent problem even when stockholders disagree about the firm's objectives, since institutional procedures guarantee a collective decision concerning management compensation. However, cooperation between principals is, in many cases, unlikely (the lack of coordination between various state and federal agencies is notorious) or impossible (when decisions are delegated to a common agent by participants in the same industry, explicit cooperation between principals is legislatively proscribed). It is then natural to think of principals as choosing incentive schemes noncooperatively, in which case the traditional bilateral agency framework is inadequate as a descriptive tool.

The literature contains few explicit models of (noncooperative) common agency. Those which do exist are tailored to highly specialized problems. Mintz and Tulkens (1984) have analyzed commodity tax competition between member

[4] In comparison, auction companies accounted for only $8 billion of wholesale trade.

[5] In each of these wholesale and retail examples, the marketing effort of the agent is typically not directly observed by the principals. Thus, asymmetric information is an important feature of these examples.

[6] The class of intrinsic common agency problems also includes a hodgepodge of special cases involving economic influence. For example, a political officeholder may allow bribes or campaign contributions to alter his legislative behavior. Such cases are distinct from the planning problems described above, in that the principals have no constitutionally guaranteed power to impose incentive schemes, and must instead bargain with the agent. These problems are beyond the scope of the current paper.

states of a federation. Bernheim and Whinston (1984) modeled the institution of common marketing agents explicitly. Baron (1984) considered noncooperative regulation of nonlocalized externalities where the objectives of distinct regulators conflict. Braverman and Stiglitz (1982) proposed a model of sharecropping in which farmers are simultaneously responsible to landlords and creditors. Stiglitz (1985) suggested a view of corporate management, in which the manager acts as an agent of both stockholders and corporate creditors.

This paper represents a first step towards developing a coherent, widely applicable, abstract framework for analyzing instances of common agency. This task is made difficult by the proliferation of highly varied institutional contexts in which common agency appears. No doubt the many problems mentioned above call for a variety of formal representations. Nevertheless, as with the literature on bilateral agency, we can hope to discern a common structure for this class of problems, and to distill the essential features of this structure, producing a simple, stylized, yet useful model of common agency.

Specifically, we envision a number of principals, who simultaneously and noncooperatively announce incentive schemes for a common agent. The agent's action is not directly observable by the principals. For concreteness, the reader might keep the following hypothetical situation in mind: several levels of government individually wish to raise revenues by taxing a firm, at the same time influencing the firm's behavior (perhaps creating incentives for effluent abatement). They face two constraints: the firm must earn nonnegative profits, and direct observation of the firm's activities is not permitted (perhaps monitoring is costly).

Our investigation primarily concerns two questions about equilibrium behavior: which action is implemented, and what incentive structure leads to its implementation? Briefly, our results can be summarized as follows. Regardless of which action is implemented in equilibrium, it is implemented efficiently—the aggregate incentive scheme induces the agent to select the equilibrium action at minimum cost. In addition, whenever collusive behavior would implement the first-best action at the first-best level of cost, noncooperative equilibrium is fully efficient (strong Nash equilibria exist, and necessarily induce efficient outcomes). Further, when this condition fails, noncooperative interaction will, quite generally, fail to produce a second-best outcome. Thus, coordination problems between principals induce further inefficiencies precisely when informational asymmetries cause welfare losses even in the presence of complete cooperation. Other related results concern the existence of equilibria, the distribution of net rewards among principals, the characteristics of actions chosen in inefficient equilibria, and potential institutional remedies for welfare losses induced by noncooperative behavior.

The paper is organized as follows. We describe a general model of common agency in Section 2. Section 3 presents a powerful result characterizing necessary and sufficient conditions for equilibrium. This result is used extensively in Section 4, where we describe the efficiency properties of equilibria. We consider the question of existence in Section 5. Section 6 closes the paper with a brief conclusion.

2. THE MODEL

Our model is a natural extension of the standard bilateral principal-agent problem to the case of many principals. Consider an environment consisting of J risk-neutral principals and a single agent. The agent may take an unobservable action that determines the probability distribution of monetary rewards received by the various principals, or alternatively, may choose not to participate at all. Should the agent participate, one of N possible outcomes—each associated with specific monetary payoffs for the various principals—results from his action choice. We denote the set of possible probability distributions over these outcomes (each induced by an agent action choice) by the set Π (a subset of R^N) and a representative element of this set by $\pi \equiv (\pi_1, \ldots, \pi_N)$. In what follows, we shall (without loss of generality) model the agent as choosing an element of Π directly.

Principal j's monetary payoffs for the various outcomes are given by the N-dimensional vector $q^j \equiv (q_1^j, \ldots, q_N^j)$. Given our assumption of risk-neutrality on the part of the principals, principal j's gross expected payoff when probability distribution π is chosen by the agent is given by $\pi \bullet q^j$ (throughout we use "$\bullet$" to denote the inner product of two vectors). Note that since $\operatorname{argmax}_{\pi \in \Pi} \pi \bullet q^j$ is not generally the same for all j, the principals typically have conflicting views about which action should be chosen. We also define the payoff vector for the J principals in aggregate as $q \equiv \sum_{j=1}^J q^j$. If the agent does not participate, then principal j receives monetary payoff Q^j (we define $Q \equiv \sum_{j=1}^J Q^j$).

In this setting each principal may have an incentive to attempt to influence the agent's distribution choice. We assume that while the agent's choice is not observable, the particular outcome realized is observed by all of the principals.[7] Thus, each principal's strategy consists of an outcome-contingent reward scheme—that is, principal j offers a compensation vector $y^j \equiv (y_1^j, \ldots, y_N^j) \in R^N$. No payments are made in the event of non-performance. In the extensive form of our multiple-principal/single-agent game, the J principals move simultaneously, each choosing an outcome-contingent reward scheme; the agent observes these offers, and then selects a distribution (or nonparticipation).

We assume that the agent, in deciding which distribution to choose, is concerned only with the *aggregate* incentive scheme offered by the principals, $y \equiv \sum_{j=1}^J y^j$. This allows us to define a set $C \subseteq \Pi \times R^N$ as follows: $(\pi, y) \in C$ if and only if the agent can be induced to pick π by the aggregate incentive scheme y.[8] In the standard agency model, for example, the agent maximizes the expected value of a von Neumann-Morgenstern utility function that is separable in income and action; the set C is defined by a set of incentive compatability constraints and a reservation utility level. As many of our results do not require this much structure, however, we shall leave the set C unrestricted and impose the standard agency formulation only when needed. For future reference we state the standard agency formulation assumption as:

[7] Note that this assumption may imply that a principal can observe more than the level of his own monetary payoff.

[8] For expositional simplicity, we have failed to explicitly state here the obvious requirement that $\pi \in \Pi$ and $y \in R^N$. Here, and in all that follows, this requirement should be understood to hold.

ASSUMPTION (A.1): *There exist real valued differentiable functions* $v(\bullet)$ *and* $g(\bullet)$ *such that* $(\pi, y) \in C$ *if and only if*: (*i*) $\pi \bullet V(y)+g(\pi) \geq 0$; (*ii*) $\pi \bullet V(y)+g(\pi) \geq \hat{\pi} \bullet V(y)+g(\hat{\pi})$ *for all* $\hat{\pi} \in \Pi$; *where* $V(y) \equiv [v(y_1), \ldots, v(y_N)]$ *and* $v(\bullet)$ *is strictly increasing.*

Without this restriction, the set C allows for a much richer class of agent preferences than in the standard agency formulation. It also permits much broader interpretations: the "agent's choice" could, for example, be the result of some complex organizational interaction for which no maximizing model is descriptive.

Finally, we assume throughout that any principal can always ensure agent nonparticipation regardless of other principals' offers. This assumption is clearly valid in the standard agency formulation of our model, since a sufficiently negative compensation offer always succeeds in driving the agent below his reservation utility level. One could easily reformulate our results without this assumption.

We wish to focus upon equilibria for our model in which agent participation occurs.[9] We say that $(\pi_0, \{y_0^j\}_{j=1}^J)$ is an *equilibrium* (with agent participation) of our multiple-principal/single-agent game if and only if

(i) (π_0, y_0^i) solves $\max_{\pi, y^i} \pi \bullet (q^i - y^i)$

subject to $\left(\pi, y^i + \sum_{j \neq i} y_0^j\right) \in C$; and,

(ii) $\pi_0 \bullet (q^i - y_0^i) \geq Q^i$, for all $i = 1, \ldots, J$.

Condition (i) requires that no principal has an alternative scheme which, when offered to the agent, would yield a higher net payoff, taking into account the agent's new choice of action.[10] Condition (ii) requires that no principal prefers the agent to choose nonperformance.

Below we shall employ a simple reformulation of condition (i). By a change of variables we obtain:

(i') (π_0, y_0) solves $\max_{\pi, y} \pi \bullet \left(q^i + \sum_{j \neq i} y_0^j - y\right)$

subject to $(\pi, y) \in C$.

This reformulation emphasizes that a principal can always compose his offer in two steps: he first undoes the offers of the other principals, and then decides upon some aggregate offer. Clearly, $(\pi_0, \{y_0^j\}_{j=1}^J)$ is an equilibrium if and only if conditions (i') and (ii) hold for all $i = 1, \ldots, J$.

[9] An equilibrium always exists for our model in which nonparticipation arises; if all but one principal offers very low compensation, the last principal is always willing to do the same, so that none find it worthwhile to encourage participation.

[10] We implicitly assume that each principal is able to resolve the agent's indifference as he wishes when deviating. Also note that our definition of equilibrium implicitly imposes the requirement of subgame perfection (Selten (1975)).

Finally, in what follows we shall be interested in comparing the outcome of this noncooperative situation with that which arises when the principals act cooperatively. A cooperative outcome, which we denote by (π^*, y^*), is of course a solution to the single principal problem:[11] $\max_{\pi,y} \pi \bullet (q-y)$ subject to $(\pi, y) \in C$.

3. GENERAL CHARACTERIZATION OF EQUILIBRIUM

We begin with a simple, yet powerful, result which characterizes necessary and sufficient conditions for an equilibrium in our model. We will say that (π_0, y_0) can be implemented in equilibrium if there exists an equilibrium, $(\pi_0, \{y_0^j\}_{j=1}^J)$, with $y_0 = \sum_{j=1}^J y_0^j$.

LEMMA 1: *$\{n_0, y_0)$ can be implemented in equilibrium if and only if*

$$\pi_0 \bullet (q - y_0) \geqslant Q \tag{1}$$

and (π_0, y_0) solves

$$\max_{\pi,y} \pi \bullet (q + (J-1)y_0 - Jy) \tag{2}$$

subject to: $(\pi, y) \in C$.

PROOF: First, suppose that (π_0, y_0) can be supported by an equilibrium, $(\pi_0, \{y_0^j\}_{j=1}^J)$. Summing condition (ii) over i immediately gives us (1). Since (π_0, y_0) satisfies (i') for each i, it also maximizes the sum of these objective functions:

$$\sum_{i=1}^J \pi \bullet \left(q_i + \sum_{j \neq i} y_0^j - y \right).$$

But this simplifies to $\pi \bullet (q + (J-1)y_0 - Jy)$, as desired.

Now suppose that (π_0, y_0) satisfies (1) and solves (2). Define

$$y_0^i = [(J-1)q^i - q^{-i} + y_0]/J + \alpha^i \mathbf{1} \tag{3}$$

where $\mathbf{1}$ is a vector of ones, the α^i's are arbitrary scalars which satisfy $\sum_{i=1}^J \alpha^i = 0$, and $q^{-i} \equiv \sum_{j \neq i} q^j$. Clearly, $\sum_{i=1}^J y_0^i = y_o$. We first wish to show that (π_0, y_0) solves (i') for all i, given these y_0^i's. Substituting, we wish to show that (π_0, y_0) solves

$$\max_{\pi,y} \pi \bullet \left(q^i + \sum_{j \neq i} [(J-1)q^j - q^{-j} + y_0]/J + \sum_{j \neq 1} \alpha^j \mathbf{1} - y \right) \tag{4}$$

subject to $(\pi, y) \in C$.

[11] There may, of course, be more than one solution to this problem. In what follows, we use (π^*, y^*) to denote any arbitrary choice from among these solutions. Results that concern (π^*, y^*) apply to all solutions.

Simplifying, we see that the maximand of (4) is equivalent to

$$\pi \bullet ([q+(J-1)y_0]/J-y)+\alpha^{-i},$$

which is identical to the objective function of (2) up to a scalar (α^{-i}), and a multiplicative constant (J). Thus, since (π_0, y_0) solves (2), it also solves (4).

Now, take $\alpha^i = Q/J - Q^i$. Clearly, $\sum_{j=1}^{J} \alpha^i = 0$. Further, it is easy to check that (ii) is now satisfied as well. So (π_0, y_0) can be implemented in equilibrium. *Q.E.D.*

Lemma 1 provides an illuminating characterization of equilibrium which facilitates comparison with the cooperative solution; we will have more to say about this comparison in the subsequent sections. The proof of Lemma 1 also has an appealing interpretation that sheds considerable light on the problem of common agency. By restating the fundamental equilibrium condition as (i′), rather than (i), we underscore the need to make principals' objectives congruent in equilibrium: since all principals can effect the same changes in the aggregate incentive scheme, none must find any such change worthwhile. One can think of this congruence as being accomplished through implicit sidepayments among principals. We say implicit because sidepayments are transferred via the agent (only the net incentive scheme matters, so each principal can take out what others put in before designing his preferred scheme). In this light, we may interpret equation (3) as follows: each principal transfers $1/J$th of his payoff to every other principal (so that his total net payment is $[(J-1)q^i/J - q^{-i}/J]$). After these transfers have been carried out, each principal's objective function is identical: $\pi \bullet (q/J - \hat{y}^i)$ ($\hat{y}^i$ is i's net-of-transfer incentive payment). It is easy to see that, once principals' preferences have been made congruent in this way, one can construct an equilibrium by dividing the aggregate optimal incentive scheme equally, and adding appropriate fees (the α^i's) to assure voluntary participation. The fees are simply set to reflect the extent to which the value of each principal's alternative opportunities differs from the mean, $(Q^i - Q/J)$.

One other aspect of this proof merits comment. Note that the choice of the α^i's is completely arbitrary, as long as they sum to zero, and are chosen so that each principals' participation constraint is satisfied. Thus, we conclude that the distribution of equilibrium net payoffs among principals is almost completely indeterminant—one can only say that i will receive at least Q^i.

4. EFFICIENCY

We now turn to an investigation of the action and incentive schemes that arise in the equilibria of our multiple-principal/single-agent game. In particular, we are interested in the question of efficiency: how do these actions and incentive schemes compare to those that would occur if the principals were to cooperate fully with one another, (π^*, y^*)?

We begin by providing a strong result concerning equilibrium incentive schemes: regardless of which action is implemented in equilibrium, the aggregate incentive scheme is necessarily the least-cost way of inducing the agent to select that action.

THEOREM 1: *Suppose that* $(\pi_0, \{y_0^j\}_{j=1}^J)$ *is an equilibrium. Then* $y_0 = \sum_{j=1}^J y_0$ *solves*

$$\min_{y \in R^N} \pi_0 \bullet y$$

subject to $(\pi_0, y) \in C.$

PROOF: Since $(\pi_0, \{y_0^j\}_{j=1}^J)$ is an equilibrium, Lemma 1 implies that

$$[q+(J-1)y_0 - Jy_0]\pi_0 \geqslant [q+(J-1)y_0 - Jy]\pi_0$$

for all y such that $(\pi_0, y) \in C$. Clearly, this implies that $y_0\pi_0 \leqslant y\pi_0$ for all y such that $(\pi_0, y) \in C$. *Q.E.D.*

The intuition behind Theorem 1 is quite straightforward. We can always view a principal as constructing his incentive scheme in two steps: he first undoes what all the other principals have offered and then makes an "aggregate" offer (this was exactly the idea behind the change of variable used above). Clearly, if we are at an equilibrium, each principal must, in this second step, select an aggregate offer that implements the equilibrium action at minimum cost.

At this point, we note another, important aspect of Lemma 1: if one can solve the *single* principal (cooperative) cost-minimization problem, it is simple to verify whether any particular distribution can arise as an equilibrium of a *multiple* principal game. To see this, define $\bar{y}(\pi)$ to be the set of cost-minimizing aggregate incentive schemes that implement π; that is,

$$\bar{y}(\pi) = \underset{y \in R^N}{\operatorname{argmin}}\ \pi \bullet y$$

subject to $(\pi, y) \in C.$

Then, condition (2) of Lemma 1 can be restated as the requirement that, for any $y(\pi) \in \bar{y}(\pi)$, π_0 solves

$$\max_{\pi \in \Pi} \pi \bullet [q+(J-1)y(\pi_0) - Jy(\pi)].$$

Having established the efficiency of the equilibrium aggregate incentive scheme *given* the equilibrium action, we now turn to an investigation of the properties of actions that *actually* arise in equilibrium. Our next result provides conditions under which the cooperative outcome can arise in an equilibrium of our noncooperative game. In it we refer to the following two special cases:

ASSUMPTION (A.2): *For any functional selection* $y(\pi) \in \bar{y}(\pi)$, *if* $y(\pi^m) \equiv [y_1(\pi^m), \ldots, y_N(\pi^m)]$ *is such that* $y_i(\pi^m) = y_j(\pi^m)$ *for all* i *and* j, *then* π^m *solves* $\min_{\pi \in \Pi} \pi \bullet y(\pi)$.

ASSUMPTION (A.3): $\bar{y}(\pi) = \bar{y}(\pi') = \bar{y}$ *for all* $\pi, \pi' \in \Pi$.

Assumption (A.2) requires that the least-cost method of assuring the agent's participation be to offer a constant ("flat") incentive scheme. This assumption is satisfied in the standard agency context [see Assumption (A.1)] with a strictly risk-averse agent.[12] Assumption (A.3) requires that there be an incentive scheme, $\bar{y}$, that is the unique cost-minimizing way to implement every agent action. This requirement is met, for example, in the standard agency context when the agent is risk-averse and has no direct preferences over his actions.

We are now in a position to state our result. Note that, under the conditions stated, (π^*, y^*) arises not only in an equilibrium, but also in a *strong* equilibrium (Aumann (1959)); *furthermore, no strong equilibrium yields any other outcome.*

THEOREM 2: *If any of the following three conditions hold, then (π^*, y^*) arises in a strong equilibrium, and no strong equilibrium yields any other outcome: (i) y^* is constant and Assumption (A.2) holds; (ii) Assumption (A.3) holds; (iii) Assumption (A.1) holds with $v(\bullet)$ a linear function. Furthermore, in case (ii), no other outcome arises in any equilibrium.*

PROOF: We first show that (π^*, y^*) is implemented by some equilibrium in each of the three cases mentioned.

(i) Setting $y_0 = y^*$ in (2), one obtains the requirement that (π^*, y^*) solve

$$\max_{\pi, y} \{\pi \bullet (q-y) - (J-1)[\pi \bullet y - k]\}$$

$$\text{subject to} \quad (\pi, y) \in C,$$

where k is the common value of each element of y^*. By definition (π^*, y^*) maximizes the first term of this expression, and by Assumption (A.2) it minimizes the second. Hence, by Lemma 1, (π^*, y^*) arises in an equilibrium of our multiple principal game.

(ii) By Theorem 1, we can rewrite condition (2) of Lemma 1 as

$$\max_{\pi \in \Pi} \pi \bullet [q + (J-1)y^* - J\bar{y}(\pi)].$$

However, under Assumption (A.3), $y^* = \bar{y}(\pi)$ for all π. Thus, the objective function of (2) becomes $\pi \bullet [q - y(\pi)]$. Clearly, π^0 solves this problem if and only if $\pi^0 = \pi^*$. (Note that this also establishes the final claim of the theorem.)

(iii) Without loss of generality we can take $v(y) = (y_1, \ldots, y_N)$. Under Assumption (A.1) any cost-minimizing incentive scheme entails the reservation utility constraint binding (see, for example, Grossman and Hart (1983))—that is, $\pi \bullet y(\pi) = -g(\pi)$ for all $\pi \in \Pi$. Furthermore, $0 \geqslant \pi \bullet y(\pi^*) + g(\pi)$ for all $\pi \in \Pi$. Thus, for all $\pi \in \Pi$, $0 \geqslant \pi \bullet [y(\pi^*) - y(\pi)]$. This implies that $\pi^* \in \operatorname{argmax}_{\pi \in \Pi} \pi[y(\pi^*) - y(\pi)]$. Therefore, (π^*, y^*) solves

$$\max_{\pi, y} \pi \bullet (q-y) + (J-1)\pi \bullet [y(\pi^*) - y]$$

$$\text{subject to} \quad (\pi, y) \in C$$

since it maximizes each term individually.

[12] With a risk-neutral agent other, nonconstant incentive schemes can yield the same implementation cost while still ensuring agent performance.

We now argue that in Cases (i)—(iii) there is a strong equilibrium which implements (π^*, y^*). Consider the strategies described in the proof of Lemma 1:

$$y_*^i = [(J-1)q^i - q^{-i} + y^*]/J. \tag{5}$$

It suffices to show that for any subset of Y principals, T, (π^*, y_*^T) solves:

$$\max_{\pi, y^T} \pi \bullet (q^T - y^T)$$

$$\text{subject to} \quad (\pi, y^T + y_*^{-T}) \in C,$$

where $y_*^T = \sum_{i \in T} y_*^i$, $y_*^{-T} = y^* - y_*^T$, and $q^T = \sum_{i \in T} q^i$. Using a change of variables, we can restate this problem as

$$\max_{\pi, y} \pi \bullet [q^T - y + y_*^{-T}] \tag{6}$$

$$\text{subject to} \quad (\pi, y) \in C.$$

Summing the incentive schemes in (5) over $i \in T$ we obtain

$$\begin{aligned} y_*^{-T} &= \sum_{i \in T} [(J-1)q^i - q^{-i} + y^*]/J \\ &= [(J-1)q^{-T} - (J-Y)q^T - (J-Y-1)q^{-T} + (J-Y)y^*]/J \\ &= \left[q^{-T} - \left(\frac{J-Y}{J}\right)q + \left(\frac{J-Y}{J}\right)y^*\right]. \end{aligned}$$

Substituting this expression into the maximand of (6) yields

$$\pi \bullet \left[\left(\frac{Y}{J}\right)q + \left(\frac{J-Y}{J}\right)y^* - y\right]. \tag{7}$$

However, (7) can be rewritten as

$$\left\{\frac{J-Y}{J(J-1)}\right\}\pi \bullet [q + (J-1)y^* - Jy] + \left\{\frac{Y-1}{J-1}\right\}\pi \bullet (q-y). \tag{8}$$

Now, $(J-Y)/J(J-1) \geq 0$ and $(Y-1)/(J-1) \geq 0$ as well. Then, since (π^*, y^*) maximizes each term of (8) independently, it solves (6) for any arbitrary group T. Thus, we have found a strong equilibrium which implements (π^*, y^*).

Finally, it is clear that no $\pi \neq \pi^*$ can be supported in any strong equilibrium, since the coalition of the whole could make a Pareto improvement. *Q.E.D.*

The three conditions stated in Theorem 2 are quite special, but nevertheless important. In the standard agency formulation (Assumption (A.1)), cooperation among the principals will implement the first-best action at first-best cost if and only if (a) the agent's most preferred action and the principals' (jointly) most preferred action are the same (condition (i)), (b) the agent has no preferences over actions (condition (ii)),[13] or (c) the agent is risk-neutral (condition (iii)). Our result states that, under these conditions, noncooperative action will also implement the first-best action at first-best cost.

One interesting corollary to this result deserves mention. If the principals are playing a zero-sum game (so that $q_1 = \cdots = q_N$), then (π^*, y^*) will solve min $\pi \bullet y(\pi)$. Thus, in the standard agency context with risk-averse agent, Theorem 3 [condition (i)] indicates that (π^*, y^*) is implemented in equilibrium.

Some insight into Theorem 2 can be gained by rewriting (8) as

$$(9) \qquad (Y/J)\pi \bullet (q - y^*) + \pi \bullet (y^* - y).$$

This expression has a natural interpretation:[14] since the equilibrium compensation offers bring the preferences of the various principals into line (as discussed above), each subgroup views changes in the first term of (9) as its gain from altering the distribution selected. The second term, on the other hand, represents its incremental costs (or savings) of accomplishing this. In each of the cases analyzed above, however, (π^*, y^*) maximizes each of these terms individually.

In fact, this sort of "term-by-term" maximization is critical for equilibrium implementation of (π^*, y^*). Since the maximand of Lemma 1 can be rewritten as

$$(10) \qquad \pi \bullet (q - y) + (J-1)\pi \bullet (y^* - y)$$

and since (π^*, y^*) maximizes the first term, with sufficient smoothness it must also locally maximize $\pi \bullet (y^* - y)$ if it is to arise in any equilibrium. This observation leads to our next result: in the standard agency context (Assumption (A.1)) with a risk-averse agent (and sufficient smoothness), whenever $V'(y^*) \equiv (v'(y_1^*), \ldots, v'(y_N^*))$ is nonconstant, (π^*, y^*) cannot arise in equilibrium. However, $V'(y^*)$ is constant only under the conditions stated in Theorem 2 (either y^* is constant, or the agent is risk-neutral). Thus, these results provide a very concise answer to the question of when noncooperative behavior by principals leads to a cooperative outcome in this context: it does so if and only if nonobservability would cause no welfare loss were the principals to cooperate fully.

In what follows it is useful to define the agent's best response correspondence (under participation):

$$\bar{\pi}(y) \equiv \{\pi \in \Pi \mid (\pi, y) \in C\}.$$

We can now formally state our result:

THEOREM 3: *Suppose that Assumption (A.1) holds, and that there exists a functional selection $\pi(y) \in \bar{\pi}(y)$ with $\pi(y^*) = \pi^*$ such that $\pi(y)$ is differentiable at y^*. If $v'(y_n^*) \neq v'(y_m^*)$ for any n and m such that $\pi_n^*, \pi_m^* > 0$, then (π^*, y^*) cannot be implemented in any equilibrium.*

[13] Under Assumption (A.1), this is a special case of condition (i); note, however, that we obtain a stronger result for (ii).

[14] Note that when $Y = 1$ we get exactly the condition of Lemma 1.

PROOF: Suppose that (π^*, y^*) were implemented in an equilibrium. Then, by Lemma 1, y^* must be a solution to

$$\text{(11)} \qquad \max_{y} \pi(y)\bullet(q-y)+(J-1)\pi(y)\bullet(y^*-y)$$

$$\text{subject to} \quad y\in\{y\in R^N \mid \text{there exists } \pi\in\Pi \text{ such that } \pi\bullet V(y)+g(\pi)\geq 0\}.$$

Without loss of generality, assume that $\pi_1^*, \pi_n^*>0$ and that $v'(y_1^*)\neq v'(y_n^*)$. Also define[15]

$$\hat{y}_n(y_1)\equiv v^{-1}\left\{-\left[\pi_1^* v(y_1)+\sum_{K\neq n}\pi_k^* v(y_k)+g(\pi^*)\right]\Big/\pi_n^*\right\}.$$

Clearly, under our assumption, $\hat{y}_n(\bullet)$ is differentiable and, in particular,

$$\text{(12)} \qquad \frac{d\hat{y}_n(y_1^*)}{dy_1}=-\frac{\pi_1^*}{\pi_n^*}\frac{v'(y_1^*)}{v'(y_n^*)}.$$

Next, consider a change in y^* that only changes compensation under outcomes 1 and n. For convenience define:

$$Y(y_1)\equiv(y_1, y_2^*, \ldots, y_{n-1}^*, \hat{y}_n(y_1), y_{n+1}^*, \ldots, y_N^*).$$

Now, note that if y^* solves (11) then y_1^* must, for some $\delta>0$, solve:

$$\text{(13)} \qquad \max_{y_1} \pi(Y(y_1))\bullet[q-Y(y_1)]+(J-1)\pi(Y(y_1))\bullet[y^*-Y(y_1)]$$

$$\text{subject to} \quad y_1\in[y_1^*-\delta, y_1^*+\delta].$$

By the definition of y^*, the derivative of $\pi(Y(y_1))\bullet[q-Y(y_1)]$ with respect to y_1 is zero at $y_1=y_1^*$. However,

$$\begin{aligned}\frac{d}{dy_1}\{\pi(Y(y_1))\bullet[y^*-Y(y_1)]\}_{y_1=y_1^*}&=-\pi_1^*+\pi_n^*\left[\frac{d\hat{y}_n(y_1^*)}{dy_1}\right]\\&=-\pi_1^*\left[1-\frac{v'(y_1^*)}{v'(y_n^*)}\right]\\&\neq 0.\end{aligned}$$

Thus, y_1^* cannot solve (13)—a contradiction to (π^*, y^*) arising in an equilibrium. *Q.E.D.*

[15] Note that if $v(\bullet)$ is bounded, $\hat{y}_n(\bullet)$ may not exist globally: however, as long as $v(\bullet)$ is strictly increasing locally around y_1^* and y_n^* it will exist locally, which is all we require.

It is important to note that smoothness of the agent's best response function is essential to the result of Theorem 3. In the standard agency context, (A.1), this condition will hold if $v(\bullet)$ and $g(\bullet)$ are strictly concave and $\pi(y^*)$ is interior to Π. If Π is finite, however, then this restriction certainly will not be satisfied. In this circumstance, if $\pi^* \bullet q - \{\max_{\pi \neq \pi^*} \pi \bullet q\}$ is sufficiently great, then it is easy to see that (π^*, y^*) can arise in equilibrium even if y^* is nonconstant. Thus, without smoothness, a second-best efficient (i.e., cooperative) outcome can arise in equilibrium, even when nonobservability would cause welfare losses to the principals were they to cooperate fully.

One example that satisfies the conditions of Theorem 3 is the well-known "effort" model (c.f., Holmstrom (1979)), in which the distribution of outcomes is determined by the effort level of the agent, $e \in \mathbb{R}$. Formally, this case is described by the following assumption.

ASSUMPTION (A.4): *Assumption (A.1) holds and, furthermore, there exists a variable $e \in \mathbb{R}$ and a differentiable function $\pi(e)$ such that*

(*i*) $$\Pi \equiv \{\hat{\pi} \mid \hat{\pi} = \pi(e) \text{ for some } e \in \mathbb{R}\}$$

and,

(ii) $$g(\pi(\hat{e})) > g(\pi(e)) \quad \text{if and only if } \hat{e} < e.$$

In such circumstances, it is natural to inquire into the relationship between the agent's equilibrium effort level and the second-best effort level. Intuition suggests that a free-rider problem is likely to exist among the principals: equilibrium requires that each principal's objectives be brought into line, but this causes each principal to weigh only $1/J$th of the gain from encouraging an increase in the agent's effort level against the full incremental costs of doing so. Our next result provides sufficient conditions under which this intuition holds. In its statement, we use $y(e)$ as short-hand for $y(\pi(e))$, the cost-minimizing aggregate scheme for implementing $\pi(e)$, and use e^* to denote the (second-best) efficient level of effort.

THEOREM 4: *Suppose that Assumption (A.4) holds and that $v(\bullet)$ is strictly concave. If, for all $e \in \mathbb{R}$,*

$$[y_n(e) - y_m(e)][y'_n(e) - y'_m(e)] > 0$$

for all $n \neq m$, then the equilibrium effort level of the agent, e_0, is such that cooperative implementation of a marginally higher effort level would raise the aggregate payoff to the principals; that is,

$$\frac{d}{de}\{\pi(e_0) \bullet [q - y(e_0)]\} > 0.$$

Furthermore, if $\pi(e) \bullet [q - y(e)]$ is concave in e, then $e_0 < e^*$.

PROOF: Application of Lemma 1 implies that if effort level e_0 arises in an equilibrium of our multiple-principal game, it must solve

$$\max_{e \in \mathbb{R}} \pi(e) \bullet [q - y(e)] + (J-1)\pi(e)[y(e_0) - y(e)].$$

Simple differentiation yields as a necessary condition that

$$\frac{d}{de}\{\pi(e_0) \bullet [q - y(e_0)]\} - (J-1)\pi(e_0) \bullet y'(e_0) = 0, \tag{14}$$

where $y'(e) \equiv [y_1'(e), \ldots, y_N'(e)]$. Next, note that in any cost-minimizing implementation, the agent's reservation utility constraint is always binding—that is,

$$\pi(e) \bullet V(y(e)) + g(\pi(e)) = 0 \quad \text{for all } e \in \mathbb{R}. \tag{15}$$

Differentiating (15) and applying the envelope theorem (since the agent is picking $\pi(e)$ to maximize this expression given $y(e)$) yields

$$\sum_{n=1}^{N} \pi_n(e) v'(y_n(e)) y_n'(e) = 0 \quad \text{for all } e \in \mathbb{R}. \tag{16}$$

Now, since $[y_n(e) - y_m(e)][y_n'(e) - y_m'(e)] > 0$ for all n and m, $y_n'(e)$ and $\{-v'(y_n(e))\}$ are similarly ordered in the sense of Hardy, Littlewood, and Polya (1952) (recall that $v(\bullet)$ is strictly concave). Hence, for all $e \in \mathbb{R}$,

$$\left\{\sum_{n=1}^{N} \pi_n(e) v'(y_n(e))\right\}\left\{\sum_{n=1}^{N} \pi_n(e) y_n'(e)\right\} > \sum_{n=1}^{N} \pi_n(e) v'(y_n(e)) y_n'(e). \tag{17}$$

But, since $v'(\bullet)$ is strictly positive, (16) and (17) imply that $\pi(e) \bullet y'(e)$ is strictly positive. Combining this result with equation (14) yields the result that

$$\frac{d}{de}\{\pi(e_0)[q - y(e_0)]\} > 0.$$

Clearly, if the expression in brackets is strictly concave, then since e^* is its maximizer, $e_0 < e^*$. *Q.E.D.*

The condition that $[y_n(e) - y_m(e)][y_n'(e) - y_m'(e)] > 0$ for all n and m holds if the efficient implementation of a marginally higher effort level entails adding larger increments of reward to those outcomes for which compensation is already high than to those for which it is low. In cases where efficient implementation schemes are monotonically related to the level of output (c.f., Milgrom (1981)), for example, such a requirement is met if the cost-minimizing incentive scheme for a high effort level is everywhere steeper than for a low one. When this condition is satisfied, encouraging a slightly higher effort level involves an increase in the risk faced by the agent, so that the expected value of this incremental compensation (evaluated at $\pi(e_0)$) is positive. Intuitively, this leads to a free-rider problem because each principal only enjoys $1/J$th of the benefits of such a change.

Finally, what can be done about the inefficiencies that can arise in situations of common agency? In particular, do institutional remedies exist that can eliminate these inefficiencies (aside, of course, from those that facilitate direct cooperation)? Two possibilities come to mind. First, as in Holmstrom (1982), the principals can restore efficiency by bringing in a risk-neutral "principals' principal" who can offer outcome-contingent payments to the principals, but cannot transact with the agent directly. It is not difficult to see that this principals' principal will find it optimal to offer payment schemes to the principals of the form

$$\beta_i \mathbf{1} - (J - 1/J)y^* + (q - q^i)$$

for $i = 1, \ldots, J$, where β_i is a constant that ensures that the net payoff to principal i is exactly Q^i. Given such schemes, (π^*, y^*) results as an equilibrium.[16]

A second possibility is to bring in a risk-neutral intermediary: the principals each individually offer outcome-contingent compensation to the intermediary, who, in turn, makes some outcome-contingent offer to the agent (the principals are proscribed from dealing with the agent directly).

Given an aggregate offer by the J principals of y, the intermediary will pick his offer to the agent, $x \in \mathbb{R}^N$, to solve,

$$\max_{\pi, x} \pi \bullet (y - x)$$

$$\text{subject to} \quad (\pi, x) \in C.$$

Clearly, for any π that the intermediary decides to implement, he sets $x = y(\pi)$.

From the principals' point of view, then, the intermediary has defined a new feasible set, $\hat{C}$, where $(\pi, y) \in \hat{C}$ if and only if

(i) $\quad \pi \bullet y - \pi \bullet y(\pi) \geq 0$

and

(ii) $\quad \pi \bullet y - \pi \bullet y(\pi) \geq \hat{\pi} \bullet y - \hat{\pi} \bullet y(\hat{\pi}) \quad \text{for all } \hat{\pi} \in \Pi.$

But note that $\hat{C}$ satisfies Assumption (A.1) with $v(\bullet)$ linear (define $\hat{g}(\pi) \equiv \pi \bullet y(\pi)$). Applying Theorem 2 implies that there exists an equilibrium (in fact, a strong equilibrium) in which the $(\hat{\pi}, \hat{y})$ that arise solve:

$$\max_{\pi, y} \pi \bullet (q - y)$$

$$\text{subject to} \quad (\pi, y) \in \hat{C}.$$

Now, note that the set $\{\pi \in \Pi \mid \text{there exists a } y \in \mathbb{R}^N \text{ such that } (\pi, y) \in \hat{C}\}$ is exactly equal to the set Π (to see this, just let $y = v(y(\pi))$ for some $y(\pi) \in \bar{y}(\pi)$). In addition, for any $\hat{\pi}$, the solution to

$$\max_{y} \hat{\pi} \bullet (q - y)$$

$$\text{subject to:} \quad (\hat{\pi}, y) \in \hat{C}$$

[16] This principals' principal could also be one of the original J principals. The important point is that he must be prevented from dealing with the agent. In such a case, the schemes offered would take the form, $\beta_i \mathbf{1} - (J - 2/J - 1)y^* + (q - q^i)$.

involves a cost of exactly $\hat{\pi} \bullet y(\pi)$ (since constraint (i) above will bind). Together, these facts imply that a second-best efficient outcome results in equilibrium—that is, that the equilibrium outcome, $(\hat{\pi}, \hat{y})$, solves

$$\max_{\pi} \quad \pi \bullet [q - y(\pi)].$$

It is worth stressing the importance of successfully preventing parties (in one case the principals' principal, in the other, the J principals) from dealing directly with the agent. As Lemma 1 indicates, any prior sidepayments among principals who *can* deal with the agent have no effect upon the equilibrium outcome.

5. EXISTENCE

Up to this point we have primarily concentrated on characterizing necessary and sufficient conditions for equilibria to be efficient in our multiple principal model. This leaves open the natural question of their existence. Theorem 2 has already resolved this question (positively) for cases in which the efficient outcome is achieved. However, for the set of cases identified in Theorem 3, we still wish to distinguish between the following two possibilities: either some inefficient action is implemented, or equilibria simply fail to exist. Unfortunately, we have not obtained any fully satisfactory proof of existence.[17] Here we provide two existence results for special cases of our model. Our first result establishes the existence of equilibria in the standard agency context with a risk-averse agent for the case where there are only two elements of the set Π. Together with Theorem 3, this implies that there are cases in which inefficient actions are selected in equilibrium (although, recall that these actions are nevertheless implemented at minimum aggregate cost).

THEOREM 5: *Suppose that $\Pi \equiv \{\pi^1, \pi^2\}$ and that Assumption (A.1) holds with $v(\bullet)$ strictly concave. Then an equilibrium exists.*

It is useful to establish the following lemma as a preliminary to proving Theorem 5.

LEMMA 2: *Suppose that $\Pi \equiv \{\pi^1, \pi^2, \ldots, \pi^M\}$ and that Assumption (A.1) holds with $v(\bullet)$ strictly concave. Then for any functional selection $y(\pi) \in \bar{y}(\pi)$,*

$$\sum_{m=1}^{M} \mu_m [\hat{\pi} - \pi^m] \bullet y(\hat{\pi}) \geqslant 0$$

where μ_m is the shadow price of the incentive constraint associated with distribution π^m in the cost-minimizing program for $\hat{\pi}$.

[17] We have also not found any counterexamples.

PROOF: Consider the cost-minimization problem:

$$\min_{y=[y_1,\dots,y_N]} \hat{\pi}\bullet y$$

subject to:

$$\text{(i)} \qquad \hat{\pi}\bullet V(y)+g(\hat{\pi})\geqslant \pi^m\bullet V(y)+g(\pi^m) \quad \text{for } m=1,\dots,M,$$

$$\text{(ii)} \qquad \hat{\pi}\bullet V(y)+g(\hat{\pi})\geqslant 0,$$

and associate Lagrange multipliers $\mu_1, \dots, \mu_M$ with the M constraints in (i), and λ with constraint (ii).

The first-order condition for $y_n(\hat{\pi})$ can be written

$$\hat{\pi}_n\left[\frac{1}{v'[y_n(\hat{\pi})]}-\lambda\right]=\sum_{m=1}^{M}\mu_m\left[\hat{\pi}_n-\pi_n^m\right]. \tag{18}$$

If we sum (18) over n we derive that

$$\lambda=\sum_{n=1}^{N}\frac{\hat{\pi}_n}{v'[y_n(\hat{\pi})]}. \tag{19}$$

We now multiply both sides of (18) by $y_n(\hat{\pi})$ and sum over n. This yields

$$\sum_{n=1}^{N}\frac{\hat{\pi}_n y_n(\hat{\pi})}{v'[y_n(\hat{\pi})]}-\lambda\hat{\pi}\bullet y(\hat{\pi})=\sum_{m=1}^{M}\mu_m[\hat{\pi}-\pi^m]\bullet y(\hat{\pi}). \tag{20}$$

If we substitute for λ from (19) we obtain:

$$\sum_{n=1}^{N}\frac{\hat{\pi}_n\bullet[y_n(\hat{\pi})-\hat{\pi}\bullet y(\hat{\pi})]}{v'[y_n(\hat{\pi})]}=\sum_{m=1}^{M}\mu_m[\hat{\pi}-\pi^m]\bullet y(\hat{\pi}). \tag{21}$$

But note that

$$\sum_{n=1}^{N}\frac{\hat{\pi}_n[y_n(\hat{\pi})-\hat{\pi}\bullet y(\hat{\pi})]}{v'[\hat{\pi}\bullet y(\hat{\pi})]}=0$$

and that $v'[y_n(\hat{\pi})]<v'[\hat{\pi}\bullet y(\hat{\pi})]$ if and only if $y_n(\hat{\pi})>\hat{\pi}\bullet y(\hat{\pi})$. Thus, the left side of (21) is nonnegative and the result is proven. *Q.E.D.*

PROOF OF THEOREM 5: Let $\Pi=\{\pi^1, \pi^2\}$ and let π^1 be a maximizer of $g(\bullet)$. If $g(\pi^1)=g(\pi^2)$ then Theorem 2 establishes existence. Thus, let $g(\pi^1)>g(\pi^2)$. By Lemma 1 an equilibrium exists if and only if at least one of the following two inequalities holds:

$$\pi^1\bullet[q-y(\pi^1)]\geqslant\pi^2\bullet[q+y(\pi^1)-2y(\pi^2)], \tag{22}$$

$$\pi^2\bullet[q-y(\pi^2)]\geqslant\pi^1\bullet[q+y(\pi^2)-2y(\pi^1)]. \tag{23}$$

Suppose neither holds. Switching the inequalities in both (22) and (23) and adding the resulting equations yields (after some simplification)

$$0<(\pi^2-\pi^1)\bullet[y(\pi^1)-y(\pi^2)]. \tag{24}$$

Clearly, $y(\pi^1)$ is such that $y_1(\pi^1) = \cdots = y_N(\pi^1)$. Thus, (24) simplifies to

(25) $\quad 0 < (\pi^1 - \pi^2) \bullet y(\pi^2).$

By Proposition 6 of Grossman and Hart (1983), however, the shadow price of the incentive constraint associated with distribution π^1 in the cost-minimizing program for implementing distribution π^2 (μ_1) is strictly positive. Applying Lemma 2 to the case of two actions implies that $(\pi^1 - \pi^2) \bullet y(\pi^2) \leqslant 0$—a contradiction to (25). Thus, at least one of the two inequalities, (22) and (23), must hold. *Q.E.D.*

While Theoreom 5 establishes the existence of equilibria in which an inefficient action arises, the assumption of only two actions is clearly quite restrictive. Our next result relaxes this assumption, but imposes others in order to establish existence.

THEOREM 6: *If there exists a continuous functional selection $y(\pi) \in \bar{y}(\pi)$ such that $\pi \bullet y(\pi)$ is convex in π and if Π is a compact and convex set, then an equilibrium exists.*

PROOF: Consider the correspondence $f: \Pi \to \Pi$ given by

$$f(\pi) = \underset{\hat{\pi} \in \Pi}{\operatorname{argmax}}\ \hat{\pi} \bullet [q + (J-1)y(\pi) - Jy(\hat{\pi})].$$

By our assumptions, the maximand is continuous in π and $\hat{\pi}$, so $f(\bullet)$ is an upper-semicontinuous correspondence. In addition, our assumption that $\hat{\pi} \bullet y(\hat{\pi})$ is convex implies that $f(\bullet)$ is convex valued. Since Π is compact and convex we can apply the Kakutani fixed point theorem to assure existence of a fixed point of $f(\bullet)$, π_0. But clearly then (π_0, y_0), with $y_0 = y(\pi_0)$, solves

$$\max_{\pi, y}\ \pi \bullet [q + (J-1)y_0 - Jy]$$

$$\text{subject to:} \quad (\pi, y) \in C,$$

and so, by Lemma 1, an equilibrium exists. *Q.E.D.*

It is worthwhile to review the assumptions of Theorem 6. The first assumption, that $y(\pi)$ is continuous, is satisfied under the standard agency Assumption (A.1) as long as the agent is risk-averse and $g(\pi)$ is strictly concave. The second assumption is that the efficient implementation cost $[\pi \bullet y(\pi)]$ is convex in π. We are, unfortunately, not aware of any general condition that would guarantee this.[18] Finally, the third assumption is that Π is compact and convex. Recall that every $\pi \in \Pi$ is implementable by some aggregate incentive scheme. Thus, these assumptions must hold for the set of *implementable* probability distributions. While compactness should typically pose no problem, convexity is far less clear.

[18] In the single principal context this condition would hold by *ex ante* randomization over incentive schemes by the principal. Note, however, that here we have restricted attention to pure strategy equilibria.

6. CONCLUSION

In this paper, we have extended the bilateral principal-agent framework to situations in which a number of risk-neutral principals independently attempt to influence the decision of a common agent. Specifically, we have provided answers to two questions. First, what are the properties of aggregate equilibrium incentive schemes? We have shown that these aggregate schemes are always efficient, in the sense that the equilibrium action is implemented at minimum cost. Second, which actions are selected in equilibrium? We have shown that whenever collusion among the principals could achieve the first-best, strong Nash equilibria exist, and necessarily induce the efficient outcome. In situations where collusion would fail to achieve the first-best, however, noncooperative interaction will (quite generally) not achieve the second-best (although noncooperative equilibria will exist as long as certain conditions are satisfied). For a special case, we have explicitly compared the noncooperative and collusive (second-best) solutions. We have also considered institutional remedies for the inefficiency which can rise through noncooperative behavior. Finally, we have shown that the distribution of net rewards among the principals is, in general, indeterminant.

At the outset, we mentioned that our model is designed for the analysis of both intrinsic and delegated common agency problems. Applications to instances of intrinsic common agency are straightforward. We have not, on the other hand, explicitly modeled processes of delegation here. While delegation complicates the analysis significantly, it does not render our model inapplicable; see Bernheim and Whinston (1985) for an explicit treatment of common marketing agents.

The principal-agent literature is, of course, not limited simply to the issue of moral hazard. A variety of authors (c.f., Harris and Raviv (1979), Mirrlees (1975)) have investigated the effects of adverse selection in similar contexts. This raises a variety of questions about preference revelation and incentive compatibility. Similarly, a natural extension of the multiple principal framework would incorporate adverse selection.

Finally, a separate branch of the principal-agent literature has studied single principal, multiple agent models (see, for example, Holmstrom (1982), Mookherjee (1984), Green and Stokey (1981), and Nalebuff and Stiglitz (1983)). An integration of this literature with our analysis of common agency might generate important insights concerning multilateral incentives in complex organizations.

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Manuscript received June, 1984; final revision received August, 1985.

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