

Comparison of Alternative Payment Mechanisms for French Treasury Auctions

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Treasury auctions around the world have been typically conducted under either the uniform-price or discriminatory format. We propose alternative payment mechanisms including the uniform-price and discriminatory formats as special cases. We compare the properties of these alternative formats in the specific context of French Treasury auctions. Our results indicate that a new payment mechanism, named the “ k^h -average-price” auction, dominates all other formats in terms of the revenues it would raise for the French Treasury. The so-called “Spanish auction” however, is found to generate the most stable stream of revenues from one auction to the next.*

I. Introduction

Treasury auctions around the world have been typically conducted under either one of two payment mechanisms: the uniform-price or the discriminatory format. Beginning with FRIEDMAN (1960), the choice between these two pricing rules has been often debated among economists.¹ Both theoretical and empirical analyses however, have yielded ambiguous results, and it appears that the ranking of the two auction formats in terms of the revenue they generate may only be established on a case-by-case basis.² For instance, in their study of 118 French Treasury auctions, ARMANTIER and SBAÏ (2006) (hereafter A&S) find that the uniform pricing rule would have generated higher revenue. The object of the present paper is to conduct a counterfactual analysis in order to explore whether alternative auction formats could have increased further the revenue raised by the French Treasury in these auctions.³

1. For surveys of the literature on Treasury auctions, see BIKCHANDANI and HUANG (1993), DAS and SUNDARAM (1996), NANDI (1997), or KLEMPERER (2000).

2. See *e.g.* BACK and ZENDER (1993), WANG and ZENDER (2002) or AUSUBEL and CRAMTON (2002) for theoretical analyses. Empirical studies based on the reduced form approach include UMLAUF (1993), TENORIO (1993), SIMON (1994), MESTER (1995), NYBORG and SUNDARESAN (1996), MALVEY and ARCHIBALD (1998), and BERG, BOUKAI and LANDSBERGER (1999); while empirical studies based on the structural approach include HORTACSU (2002), FEVRIER, PREGET and VISSER (2004), CASTELLANOS and OVIEDO (2005), as well as ARMANTIER and SBAÏ (2006).

3. Although other criteria are often considered of importance by practitioners (*e.g.* participation, volatility, or market failures such as “short squeeze”), Treasury auction formats have been typically compared in the economic literature on the basis of the revenue they generate. Accordingly, although we will consider the issue of volatility, our analysis concentrates mostly on comparing the revenues generated by different auction formats.

* JEL : D44, G28, D63 / KEY WORDS: Auctions, Treasury Auctions, Security Emission.

At a Treasury auction, a specific type of security is sold to several accredited financial institutions. The bidders submit simultaneously a sealed bid consisting of a demand schedule. A bid therefore specifies the number of units of the security requested at each possible price. The market-clearing price, also known as the stop-out-price, matches aggregate demand with the available supply of security. As previously mentioned, two basic payment mechanisms have been typically employed in practice: under the discriminatory format, the format used by most Treasuries around the world, and in particular, the French Treasury, the highest bids are filled at the bided price until supply is exhausted; under the uniform-price format, bidders pay the stop-out-price for all units they requested at prices exceeding the stop-out-price. These two auction formats, however, are not the only payment mechanisms to have been adopted for the sale of Treasury securities. For instance, the Spanish Treasury has also implemented since January 1987 the so-called “Spanish auction” to sell its Treasury securities.⁴ Moreover, hybrid formats, such as the “ α -auction” of WANG and ZENDER (2002), have recently been proposed in theoretic models.

In the present paper, we investigate the properties of these alternative formats, and we propose several new payment mechanisms. To facilitate the analysis, we loosely partition the auction formats into either a “discriminatory-type” or a “uniform-type”. As further explained in section 4, the highest price paid for a unit of the security is the same for each bidder at “uniform-type” auctions, while it differs across bidders at “discriminatory-type” auctions. The different pricing rules may be further differentiated on the basis of the factors influencing the highest price paid by each bidder for a unit of the security. Indeed, we will see that under our “ α -discriminatory” and “ α -uniform” auction formats the highest price paid by a bidder depends on the entire bid functions submitted; under our “ α -price-discriminatory” and “ α -price-uniform” auction formats the highest price paid depends only on the highest prices submitted; under the “ k^{th} -average-price” auction format only the average price of the winning bids matters; finally, at a “Spanish auction” bidders do not pay more than the weighted average price of all winning bids.

As previously mentioned, auction formats may only be ranked on a case-by-case basis. In fact, AUSUBEL and CRAMTON (2002) conclude their theoretical analysis by stating: “determining the better pricing rule is necessarily an empirical question.” In other words, the superiority of an auction format depends on specific empirical factors, such as the bidders’ distribution of types, and/or their levels of risk aversion. Therefore, to compare the pricing rules just mentioned, we concentrate on the case of French Treasury auctions. More specifically, we conduct a counterfactual analysis based on the structural parameters estimated by A&S for two specific securities sold at French Treasury auctions, to determine which pricing rule would have generated the highest revenue. As a consequence, the conclusions reached in the present paper are specific to these French Treasury auctions, and may not immediately extend to other Treasury markets in different countries. Nevertheless, the methodology we developed allows one to compare the new auction formats we propose in this paper for any Treasury auction around the world.

When we concentrate on the three payment mechanisms currently employed, we find that the revenue of the French Treasury would be higher under the uniform-price format than under the Spanish format, while it is the lowest under the discriminatory format, the payment mechanism

4. See ALVAREZ and MAZON (2002), as well as ALVAREZ, MAZON and CERDA (2003) for theoretical analyses of the Spanish auctions. See also ABBINK, BRANDTS and PEZANIS-CHRISTOU (2006) for a laboratory experiment.

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currently used by the French Treasury. When compared with alternative formats, we find that a new payment mechanism, namely the k^{th} -average-price format, dominates all others. Finally, the “Spanish auction” is found to be the most stable, in the sense that the revenue generated by the French Treasury would have been less volatile from one auction to the next.

The paper is structured as follows: section II briefly describes the market for the French Treasury securities; in section II, we summarize the structural model used by A&S, as well as their estimation results; section IV introduces the alternative payment mechanisms; the results of the counterfactual analysis are presented in section V; finally, section VI concludes.

II. The French Treasury Auction Market

Following A&S, we concentrate on two types of securities, the OAT (Obligations Assimilables du Trésor) and the BTAN (Bons du Trésor à Intérêt Annuel), sold by the French government at traditional discriminatory auctions. The OAT are the government’s long-term debt instruments with maturities ranging from seven to thirty years. The BTAN represent medium-term government debts with a maturity of either two or five years.

The timing of these auctions unfold as follows: auctions for OAT and BTAN are held respectively the first and the third Thursday of each month. Four business days before the auction, the “Agence France Trésor”, which is in charge of conducting the auction, announces the details of the different “lines” to be auctioned. A line consists of either an OAT or a BTAN with specific characteristics including the nominal yield, the maturity, as well as a bracket for the volume of security to be served. Part of the announced Treasury security may then be traded on a primary (or “when-issued”) market until the date of the auction by a limited number of authorized dealers.

Competitive bidders may submit a bid consisting in a demand function. A quantity demanded by a competitive bidder is in fact an amount in Euros representing a share of the quantity sold by the Treasury. Prices are expressed as a percentage (formulated with two decimal points) of the nominal value of the security (one Euro). Moreover, pre-qualified bidders may submit a non-competitive offer for any line. This offer consists in a (limited) amount that will be systematically served after the auction at a price equal to the (quantity weighted) average price of the awarded competitive bids. Bids by eligible institutions for all lines to be auctioned that day must be submitted either electronically or in sealed envelopes at least ten minutes prior to the auction.

Before the bids are observed, the French Treasury sets the exact quantity that will be supplied.⁵ Competitive bids are then ranked in descending order, and the stop-out-price is determined in such a way that the aggregate competitive and non-competitive demand matches the exact quantity supplied. Auction results are announced within five minutes after the end of the auction, and the Banque de France completes the delivery-versus-payment orders with the auction winners within three business days. The security may then be traded to the general public on a secondary market.

5. The Agence France Trésor has been known to set the exact quantity to be supplied as a response to exogenous short term shocks. Therefore, in contrast with the assumptions in the theoretic analysis of BACK and ZENDER (2001), the French Treasury is not suspected to act strategically when setting the quantity.

Although occasional bidders may participate, the French State's policy issuance essentially relies on a network of primary dealers (aka "Spécialistes en Valeurs du Trésor"). The role of these primary dealers is to advise and assist the French Treasury in marketing appropriately its debt. In particular, the primary dealers must be active on the primary market, and maintain a liquid secondary market.⁶ During the period studied by A&S (1998 to 2000), the primary dealers were composed of 19 institutions accounting for over 90% of the securities bought. The involvement of the primary dealers to each French Treasury auction may vary notably between financial institutions. In particular, the Agence France Trésor identified five large financial institutions (Crédit Agricole, Deutsch Bank, BNP-Paribas, Morgan Stanley, and Société Générale) who participated in most auctions and were allocated more than 60% of the securities issued during our sample period.

Finally, note that most of the bonds purchased at French Treasury auctions are eventually resold on the secondary market, or directly to the bidders' own clients (*e.g.* mutual funds, insurance companies, individuals).⁷ Moreover, the flow of pre-auction orders submitted by their own clients, may lead the auction's participants to form different forecasts about the future market value of the bond. In this context, and as further argued in A&S, the common-value paradigm may be considered the most reasonable.⁸

III. Baseline Model

III.1 *A Share Auction Model with Asymmetric Bidders and Risk Aversion*

We now present the structural model estimated by A&S. This model generalizes the common-value share auction model with random supply of WANG and ZENDER (2002), by accounting for possible informational and risk aversion asymmetries across bidders.

At a given auction, a specific quantity of a perfectly divisible good is for sale to N competitive bidders ($N \geq 2$) each maximizing his ex-ante expected utility.⁹ A bidder's decision to participate in the auction (*i.e.*, to submit a competitive bid) is assumed to be exogenous and common knowledge. The quantity supplied to the bidders by the auctioneer is unknown at the time of the auction, and it is represented by a random variable $Q \in \Theta_Q$, with cumulative distribution function (c.d.f.) $G(Q)$.¹⁰ The actual value of the good, $V \in \Theta_V$, is random with a c.d.f. $F_0(V)$. This true value is assumed to be the same to each bidder, but unknown at the time of the auction.

6. Since 2003, a "Spécialiste en Valeurs du Trésor" is also required to account for at least 2% of the volume auctioned over the last 12 months in order to maintain its status.

7. In particular, unlike the Turkish Treasury auctions analyzed by HORTACSU (2002), the primary dealers are not required by the French Treasury to hold a portion of their reserves in the form of Treasury bonds.

8. See also the argument in FEVRIER *et al.* 2004 who conclude as well that the common-value paradigm best fits the French Treasury auctions.

9. Non-competitive bids are assumed to result from exogenous decisions made prior to the auction. Therefore, non-competitive bidding will not be modelled, and it is assumed to affect only the quantity available to competitive bidders. The competitive bidders will be referred, in the remainder, as the "bidders", the "players", or the "participants".

10. As previously mentioned, the quantity available to competitive bidders at the time they submit their bids is uncertain in the French Treasury auctions because i) non-competitive bids are only revealed after the auction, and ii) the French Treasury announces only a bracket for the total quantity of securities to be supplied.

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As further explained in section 3.2, the participants in French Treasury auctions may be divided in two distinct groups. Let us denote by N_1 and N_2 ($N_1 + N_2 = N$) the number of bidders from each group participating in the auction.¹¹ Bidders within a given group are symmetric, but bidders are asymmetric across groups. Bidder i in group $l = 1, 2$ receives a signal, $s_{i,l} \in \Theta_S$ containing some private information about the value of the good. This signal is generated from a conditional distribution with c.d.f. $F_l(s_{i,l} | V)$. After bidder i in group l receives the private signal $s_{i,l}$ she submits a sealed bid. This bid consists of a schedule specifying the share of the good demanded $\varphi_{i,l}(p, s_{i,l})$ for any price $p > 0$. The demand schedules are assumed to be (piecewise) continuously differentiable. The stop-out-price p^0 is defined as the non-negative price at which aggregate competitive demand $\Phi(\cdot)$ equals total supply:

$$\Phi(p^0, s) = \sum_{i=1}^{N_1} \varphi_{i,1}(p^0, s_{i,1}) + \sum_{j=1}^{N_2} \varphi_{j,2}(p^0, s_{j,2}) = Q \quad \text{given } p^0 \geq 0 \quad (1)$$

Winning bids are those submitted for prices greater than the stop-out-price. In other words, bidder i in group l receives a quantity $\varphi_{i,l}(p^0, s_{i,l})$. At this point, it is important to note that the characterization of the stop-out-price p^0 , and the allocation process just described is common to every auction format we will be discussing in the present paper. The only difference between the auction formats will reside in the pricing mechanism applied to bidders in order to pay for the share they receive.

Although the French Treasury auctions are actually conducted with the traditional discriminatory pricing rule, we adopt the general notation of VISWANATHAN and WANG (2000) in order to introduce simultaneously the traditional uniform-price mechanism. For the quantity $\varphi_{i,l}(p^0, s_{i,l})$ received, a bidder i in group l is asked to pay

$$p^0 \varphi_{i,l}(p^0, s_{i,l}) + \alpha \int_{p^0}^{p_{i,l}^{\max}} \varphi_{i,l}(p, s_{i,l}) dp \quad (2)$$

where $p_{i,l}^{\max}$ is the highest price for which the share demanded by bidder i in group l is strictly positive. Within this formulation, $\alpha = 0$ corresponds to the price paid under the uniform-price format, and $\alpha = 1$ corresponds to the price paid under the discriminatory format. In this context, we will subsequently refer to the quantity $\int_{p^0}^{p_{i,l}^{\max}} \varphi_{i,l}(p, s_{i,l}) dp$ as bidder i 's "discriminatory surplus".

To illustrate how the traditional discriminatory and uniform-price mechanisms work, consider the following example.

Example 1: Consider an auction with two bidders (e.g., one in each group), in which the auctioneer sells one unit of fully divisible Treasury bonds. We plotted in FIGURES 1.1 and 1.2 two possible bid functions for bidders 1 and 2. First, observe that at the stop-out-price p^0 , the aggregate quantity demanded and the quantity supplied are both equal to 1. The quantity allocated to bidder 1 (respectively, bidder 2) corresponds to $q_1 = \varphi_1(p^0, s_1)$ (respectively,

11. In A&S data, N_1 , N_2 , and N vary from one auction to the next.

$q_2 = \varphi_2(p^0, s_2)$). To receive the allocated quantity $q_1 = 3/5$ (respectively, $q_2 = 2/5$), bidder 1 (respectively, bidder 2) must pay under the uniform-price auction a total price corresponding to the area denoted 1 in FIGURE 1.1 (respectively, 1.2). In contrast, to receive its allocated quantity, a bidder under the discriminatory format must pay the total price under the uniform-price format (area 1), plus his discriminatory surplus (area 2).

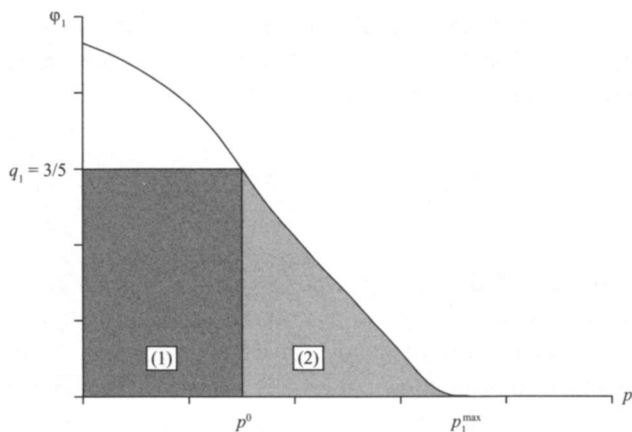


FIGURE 1.1. — Baseline Share Auction Model (Bidder 1)

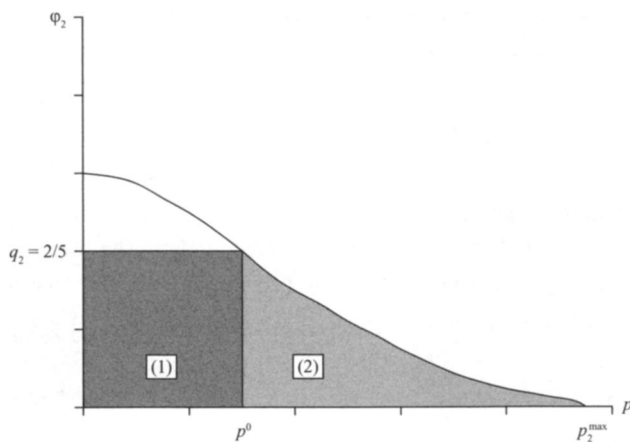


FIGURE 1.2. — Baseline Share Auction Model (Bidder 2)

Under this general pricing rule, the conditional profit of bidder i in group l may then be written as

$$\Pi_{i,l}(\varphi_{i,l}(\cdot), p^0, V, s_{i,l}) = (V - p^0)\varphi_{i,l}(p^0, s_{i,l}) - \alpha \int_{p^0}^{p_{i,l}^{\max}} \varphi_{i,l}(p, s_{i,l}) dp \quad (3)$$

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Finally, A&S find that a bidder i in group l exhibits risk aversion in the form of a CARA utility function:

$$U_{i,l}(\varphi_{i,l}(\cdot), p^0, V, s_{i,l}, \lambda_l) = -\exp[-\lambda_l \Pi_{i,l}(\varphi_{i,l}(\cdot), p^0, V, s_{i,l})] \quad (4)$$

where $\lambda_l > 0$ is the constant level of absolute risk aversion for players in group l .

To conclude this section, note that the asymmetric share auction model we just presented cannot be solved analytically. Therefore, to estimate the structural parameters, A&S rely on the Constrained Strategic Equilibrium (hereafter CSE) technique developed by ARMANTIER, FLORENS and RICHARD (2008) to approximate intractable Bayesian Nash Equilibria. This approximation technique will also be used in section 5 to solve the Bayesian Nash equilibrium and analyze the properties of the alternative auction formats.

III.2 *Estimated Structural Parameters*

A&S estimated the structural model presented in section 3.1 with a sample consisting of 118 auctions (60 OAT and 58 BTAN) which took place at 64 different dates between May 1998 and December 2000. For each auction, the Agence France Trésor provided us with i) the security pre-announced characteristics (*i.e.*, the nominal yield, the maturity, whether the line is an OAT or a BTAN, and the upper and lower bounds of the bracket for the quantities served to the competitive and noncompetitive bidders); ii) the number of bidders in each group; iii) the quantity demanded for all prices by each individual bidder; and iv) the auction's outcomes (*i.e.*, the stop-out-price, and the quantity served to each bidder). Note that the sample provided to us, although anonymous, partitions the participants in two homogenous groups of "large" or "small" financial institutions.¹² The structural model is estimated by A&S with a sample of 40,496 observations, each corresponding to a price-quantity pair on the bid function submitted by winning and non-winning bidders.¹³

We present in TABLE I summary statistics for the variables in our sample. Participation in French Treasury auctions is dominated by smaller financial institutions (14.6 small banks versus 4.5 large banks). Although the number of bidders within groups may vary, the average number of participants (roughly 19) is rather stable across auctions and lines. Large banks received a larger share of the security (63.8% on average). This result is consistent with the fact that, on average, large banks submit higher prices for the initial units demanded (101.193 versus 100.922 for small banks), and pay slightly more per unit awarded (101.185 versus 100.180). Note also that large banks are almost guaranteed to receive some of the securities issued. In contrast, a small bank is only awarded a share of the security 64.3% of the time on average. Finally, observe that the French Treasury raised a total of 186.7 billion Euros during the 118 auctions in our sample.

12. During the period covered by our sample a participant always belonged to the same group of either large or small financial institutions.

13. A&S's estimation strategy takes full advantage of each point on the step demand functions actually submitted by the bidders. In other words, A&S consider every price at which a share is demanded (*i.e.*, the points at which the step demand function strictly decreases), but they also account for all possible prices for which a bidder did not ask for an additional share (*i.e.*, the prices at which the step demand function remains flat). This latter set of data points should not be ignored, as it also contains information about a bidder's strategy. Indeed, such observations indicate that at a given price the bidder did not wish to modify the quantity she demanded.

TABLE I. — SUMMARY STATISTICS
(Average per auction unless mentioned otherwise)

		Total	OAT	BTAN
Number of auctions		118	60	58
	Total	19.025 (0.797)	18.983 (0.871)	19.069 (0.686)
Number of Competitive Bidders	Large Banks	4.508 (1.443)	4.800 (1.542)	4.207 (1.237)
	Small Banks	14.568 (1.536)	14.217 (1.603)	14.931 (1.366)
	Yield	4.826 (1.186)	5.492 (1.191)	4.138 (0.679)
	Maturity (in days)	3,541.627 (3,462.798)	5,746.683 (3,391.702)	1,260.534 (570.006)
	Quantity Supplied (in Million Euros)	1,559.500 (959.273)	1,535.544 (1,044.719)	1,584.283 (816.756)
	Non Competitive Bids (in Million Euros)	44.122 (59.100)	47.640 (63.302)	40.611 (57.610)
	Stop-Out-Price *	101.163 (6.975)	102.476 (8.898)	99.804 (3.738)
Average Price Paid Per Unit Purchased *	Large Banks	101.185 (5.985)	102.508 (6.130)	99.816 (3.401)
	Small Banks	101.180 (12.482)	102.501 (11.310)	99.813 (3.774)
Percentage Awarded to Large Banks	Total	0.638 (0.006)	0.645 (0.006)	0.625 (0.005)
	Per Bank	0.167 (0.121)	0.168 (0.096)	0.166 (0.098)
Percentage Awarded to Small Banks	Total	0.362 (0.007)	0.355 (0.008)	0.375 (0.006)
	Per Bank	0.024 (0.042)	0.024 (0.048)	0.025 (0.041)
Probability to be Awarded a Share	Large Bank	0.988	0.980	0.994
	Small Bank	0.643	0.645	0.640
Highest Price Submitted*	Large Banks	101.193 (6.700)	102.519 (8.227)	99.821 (2.403)
	Small Banks	100.922 (11.032)	102.360 (13.295)	99.419 (3.206)
Per Auction Revenue for Treasury (Million Euros)	Total	1,582.619 (664.794)	1,579.370 (535.822)	1,585.981 (473.006)
	From Large Banks	989.652 (424.910)	994.441 (384.113)	984.697 (441.201)
	From Small Banks	592.967 (293.821)	584.929 (285.573)	601.283 (298.933)
Total Revenue for Treasury		186,749.153	94,762.249	91,986.904

* Prices and revenues are expressed as a percentage of one Euro. Standard deviations are in parenthesis.

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We now briefly summarize A&S estimation results. Remember that these estimated structural parameters, along with the exogenous variables in our sample, will be used in section 5 to conduct the counterfactual analysis. A&S estimate that the quantity supplied by the French Treasury at a given auction follows the following relationship:

$$Q_t = 1,426.9 + 0.681\text{MeanBracket}_t - 0.193\text{Maturity}_t - 376.2\text{Yield}_t + 1,208.7\text{Line}_t + v_t \quad (5)$$

where $\text{MeanBracket}_t = (\underline{Q}_t + \bar{Q}_t) / 2$; \bar{Q}_t and \underline{Q}_t are the upper and lower bounds of the quantity bracket announced by the French Treasury before auction t ; Maturity_t and Yield_t are respectively the maturity and the nominal yield associated with the security sold at auction t ; Line_t is a dummy variable equal to 1 when the line is an OAT, and 0 when the line is a BTAN; and finally, v_t is an identically and independently normally distributed error term with mean zero and estimated standard deviation 475.207.

The true value of the security V_t is found to be normally distributed with mean $\mu_{V_t} = 81.165 + 4.782\text{Yield}_t - 7.223E^{-4}\text{Maturity}_t$, and standard deviation 0.632. The distribution of the private signals for bidders in group 1 (respectively, group 2) is normal with mean V_t and standard error 0.070 (respectively, 0.178). Finally, the levels of absolute risk aversion are estimated at respectively $5.732E^{-8}$ and $6.907E^{-6}$ for banks in groups 1 and 2.¹⁴ In other words, the structural parameters estimated in A&S indicate that, in contrast with group 1, group 2 consists mostly of smaller financial institutions, characterized by a higher level of risk aversion, and receiving significantly noisier signals about the true value of the security.

Once the structural parameters have been estimated, A&S conduct a counterfactual analysis to compare the traditional discriminatory and uniform-price auction formats. They find that over the 118 auctions in their sample, the French Treasury would have accumulated an additional 8.4 billion Euros, a significant increase of 4.5%, had it used the uniform-price instead of the discriminatory format. In section 5, using the same estimated parameters and the same exogenous variables, we will conduct additional counterfactual analyses to explore whether alternative auction formats could have further increased the revenue of the French Treasury during these auctions.

IV. Alternative Auction Formats

In this section, we extend the general model proposed in section 3 by specifying alternative payment mechanisms. As previously explained, the allocation mechanism remains unaffected under these alternative auction formats. In particular, the stop-out-price p^0 is still characterized by equation (1), and the security is still divided among participants submitting bids above p^0 . In other words, for a given bid function, a bidder receives the same share of the security under every auction format, and only the amount he is required to pay will differ.

14. As further discussed in A&S, the estimated absolute risk aversion parameters may appear rather low. However, the relative risk aversion levels range between 0.02 and 0.9 when calculated with the banks' profits. These figures, although still low since they do not include the entire wealth of the participants, appear more reasonable.

To start, let us rewrite equation (2), which characterizes the payment of a bidder i when he receives a quantity $\varphi_i(p^0, s_i)$, as

$$p^0 \varphi_i(p^0, s_i) + \int_{p^0}^{\bar{p}_i} \varphi_i(p, s_i) dp \tag{6}$$

where \bar{p}_i is the highest price paid by bidder i for a unit of the security.¹⁵ As previously mentioned, we partition the different payment mechanisms in two classes. The highest price paid for a unit of the security is the same across bidders under “uniform-type” auctions (*i.e.*, $\bar{p}_i = \bar{p}_j = \bar{p}$). In contrast, each winning bidder i pays a different highest price under “discriminatory-type” auctions (*i.e.*, $\bar{p}_i \neq \bar{p}_j$).

As we illustrate in example 2 below, the pricing rule in equation (6) may also be interpreted as follows: a bidder must pay his bid for any winning bid he announced at a price below \bar{p}_i , but he only needs to pay \bar{p}_i for all winning bids he submitted at prices above \bar{p}_i . Moreover, observe that this formulation includes as special cases the traditional discriminatory and uniform pricing rules, when $\bar{p}_i = p_i^{\max}$, and $\bar{p}_i = \bar{p} = p^0$ respectively. As we shall see in section 5, we find that such a partition of auction formats provides a convenient framework to organize the results of the counterfactual analysis.

IV.1 Discriminatory-Type Auctions

IV.1.1. The α -Discriminatory Auction

The α -discriminatory payment mechanism was in fact introduced in equation (2). Indeed, we can generalize the uniform-price format ($\alpha = 0$) and the discriminatory-price format ($\alpha = 1$) to allow α to vary between 0 and 1. This payment mechanism was initially suggested by VISWANATHAN and WANG (2000), and WANG and ZENDER (2002). Observe that the payment of a winning bidder in an α -discriminatory auction lays between the amount paid under the uniform and discriminatory formats. In fact, α represents the fraction of the discriminatory surplus $\int_{p^0}^{p_i^{\max}} \varphi_i(p, s_i) dp$ extracted from a bidder i , in addition to his payment under the uniform-price format.

To be consistent with equation (6), we can re-write the payment mechanism (2) as

$$p^0 \varphi_i(p^0, s_i) + \alpha \int_{p^0}^{p_i^{\max}} \varphi_i(p, s_i) dp = p^0 \varphi_i(p^0, s_i) + \int_{p^0}^{\bar{p}_i} \varphi_i(p, s_i) dp$$

where \bar{p}_i , the highest price paid by bidder i for a unit of the security, is defined as a function of the α selected by the auctioneer. Observe also that the α -discriminatory payment mechanism indeed belongs to the family of “discriminatory-type” auctions, since \bar{p}_i differs across bidders. We now illustrate how the payment mechanism may be implemented.

Example 2: Consider the same auction as in Example 1, except that the auctioneer now selects the α -discriminatory payment mechanism with $\alpha = 2/3$ (see FIGURES 2.1 and 2.2). To receive his winning share $3/5$, bidder 1 must pay a total price corresponding to the areas denoted 1 and 2. Note that this amount is in between the payment bidder 1 would have made under the uniform-price format

15. To ease the presentation, we drop the group index l in this section.

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(area 1) and the discriminatory format (areas 1, 2 and 3).¹⁶ Area 2 represents the share α ($2/3$ in this case) of the bidder's discriminatory surplus (which in FIGURE 2.1 corresponds to the sum of areas 2 and 3). FIGURE 2.2 illustrates a different but equivalent way to interpret the α -discriminatory payment mechanism. Indeed, the payment of bidder 2 for his share $2/5$ of the security may be decomposed in two areas. Area 1 indicates that, analogously with a uniform-price auction, the highest price paid by bidder 2 for a share between 0 and $q'_2 = 0.23$ is \bar{p}_2 (since for these shares he submitted a bid above \bar{p}_2). In contrast, area 2 indicates that, analogously with a discriminatory auction, he pays the price he announced (which is below \bar{p}_2) for any share between $q'_2 = 0.23$ and $q_2 = 2/5$.

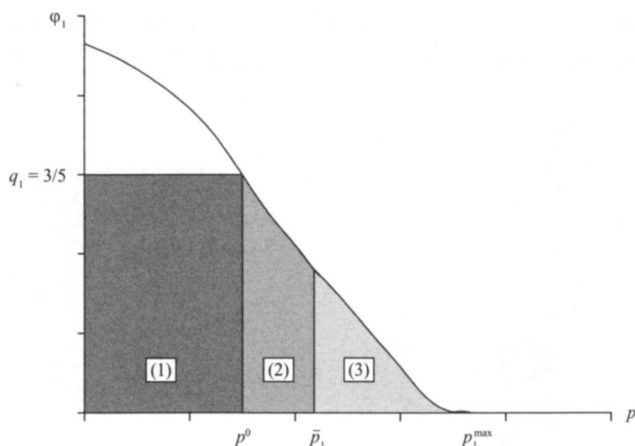


FIGURE 2.1. — α -Discriminatory Auction (Bidder 1)

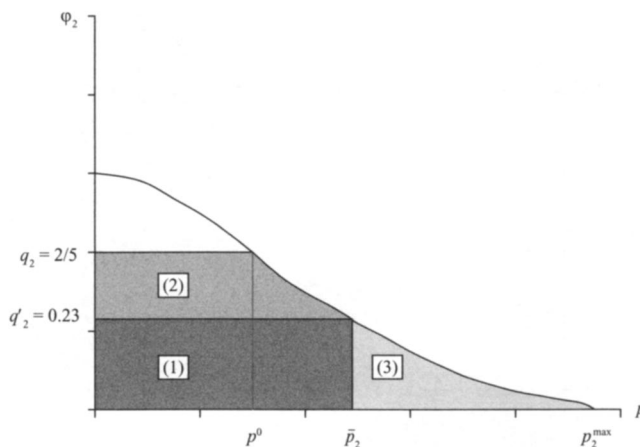


FIGURE 2.2. — α -Discriminatory Auction (Bidder 2)

16. Observe that the bid functions of bidder 1 (respectively, 2) are the same in FIGURES 1.1 and 2.1 (respectively, 1.2 and 2.2). Indeed, when comparing the different auction mechanisms throughout section 4, we assume that each bidder plays the same strategy. However, we will see in section 5 that, since the strategic environment differs, the equilibrium behavior of a bidder will be different depending on the payment mechanism.

IV.1.2. The α -Price-Discriminatory Auction

The α -Price-Discriminatory format is similar to the payment mechanism we just introduced, except that \bar{p}_i is now defined as a convex combination between p^0 , the stop-out-price, and p_i^{\max} , the highest price announced by bidder i :

$$\bar{p}_i = p^0 + \alpha(p_i^{\max} - p^0) \tag{7}$$

Note that once again, \bar{p}_i depends on the value of the parameter α selected by the auctioneer. The main difference with the α -discriminatory auction resides in the fact that the highest price paid \bar{p}_i in (7) is not immediately affected by the shape of the bid function φ_i . Indeed, \bar{p}_i depends directly on p_i^{\max} , the highest price announced by bidder i , while it only depends indirectly on the entire

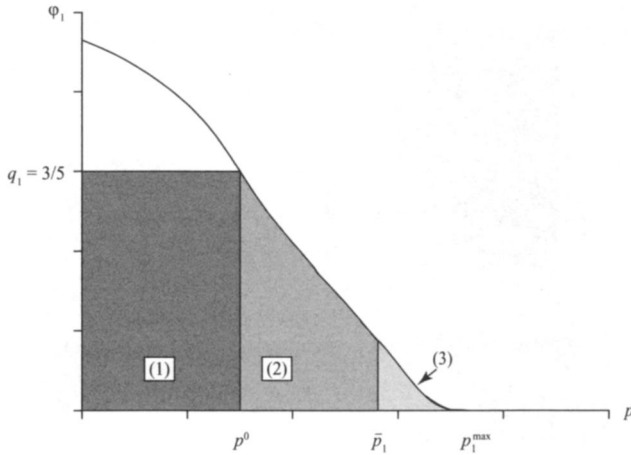


FIGURE 3.1. — α -Price-Discriminatory Auction (Bidder 1)

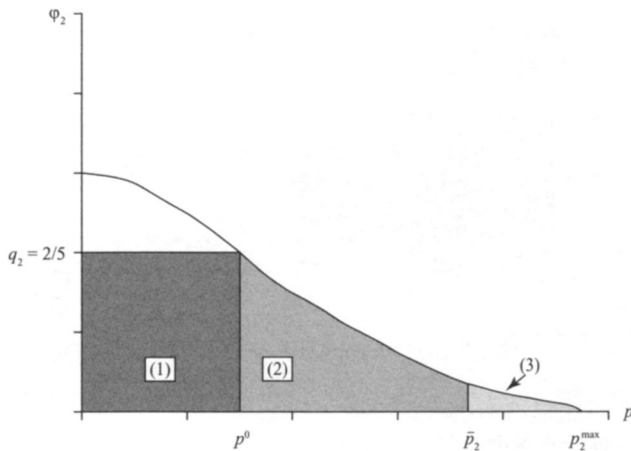


FIGURE 3.2. — α -Price-Discriminatory Auction (Bidder 2)

bid function φ_i through the determination of the stop-out-price p^0 (see equation 1). The distinction between the two auction formats may seem minor at first glance, but as we shall see, it turns out to have a substantial impact on the equilibrium bid functions. To illustrate the distinctive features of this payment mechanism, we now present an example.

Example 3: Consider the same auction as in Example 1, except that the auctioneer now selects the α -price-discriminatory payment mechanism with $\alpha = 2/3$ (see FIGURES 3.1 and 3.2). In contrast with the α -discriminatory format, \bar{p}_i , the highest price paid by bidder i , is now located at exactly $2/3$ of the distance between p^0 , the stop-out-price, and p_i^{\max} , the highest price announced by bidder i . The α -price-discriminatory payment mechanism is then such that bidder i must pay his bid for any winning share he asked at a price below \bar{p}_i , and he must pay \bar{p}_i for any winning share asked at a price above \bar{p}_i . In other words, to receive his winning share of the security $3/5$ (respectively, $2/5$), bidder 1 (respectively, bidder 2) must pay a total price corresponding to the areas denoted 1 and 2 in FIGURE 3.1 (respectively, 3.2). The payment of bidder i therefore still lays between the payments made under the traditional uniform-price (i.e., area 1) and discriminatory (i.e., areas 1, 2, and 3) auctions. Observe however, that unlike the α -discriminatory format, area 2 does not correspond anymore to $2/3$ of the discriminatory surplus (i.e., areas 2 and 3).

IV.2 Uniform-Type Auctions

IV.2.1. The α -Uniform Auction

The α -uniform payment mechanism is similar to its discriminatory counterpart presented in section 4.1.1, except that each winning bidder pays the same highest price \bar{p} . We can therefore write the payment mechanism in the α -uniform auction as

$$p^0 \varphi_i(p^0, s_i) + \int_{p^0}^{\bar{p}} \varphi_i(p, s_i) dp$$

where the common highest price \bar{p} is characterized by the following relationship

$$\int_{p^0}^{\bar{p}} \Phi(p, s) dp = \alpha \int_{p^0}^{p_i^{\max}} \Phi(p, s) dp \quad (8)$$

$\Phi(p, s)$ represents the aggregate demand as defined in equation (1), and $p_i^{\max} = \max(p_i^{\max})$ is the highest price announced across all bidders. Observe also that $\int_{p^0}^{p_i^{\max}} \Phi(p, s) dp$ may be interpreted as the aggregate discriminatory surplus. In other words, the parameter α now represents the fraction of the aggregate discriminatory surplus extracted from the bidders in addition to the revenue generated under the traditional uniform pricing rule.

The α -uniform and α -discriminatory formats share a number of characteristics: first, setting $\alpha = 0$ (respectively, $\alpha = 1$) yields in both cases the traditional uniform-price (respectively, discriminatory) payment mechanism; second, for a given value α (and the same bid functions), both mechanisms generate the same revenue for the Treasury; and third, α represents in both cases the same fraction of the aggregate discriminatory surplus. Note however that, in contrast with the α -discriminatory format, the total payment of some bidders may be the same under the α -uniform and the traditional discriminatory format. Such an event occurs for any bidder i , if his highest bid

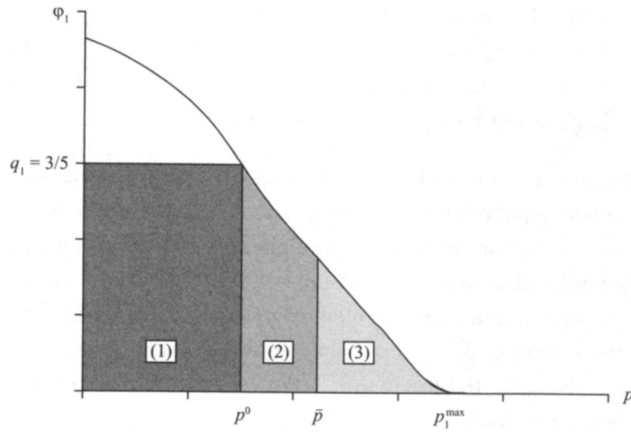


FIGURE 4.1. — α -Uniform Auction (Bidder 1)

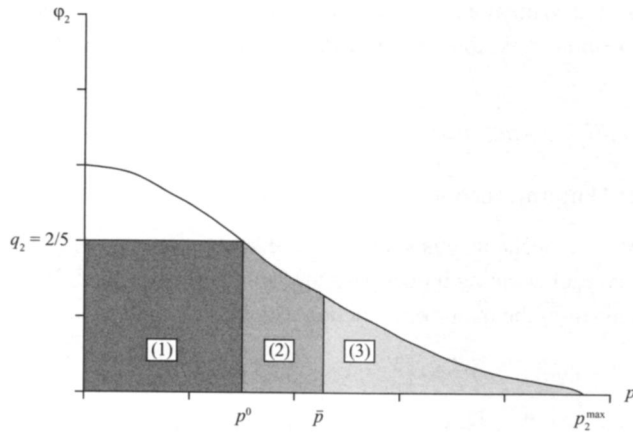


FIGURE 4.2. — α -Uniform Auction (Bidder 2)

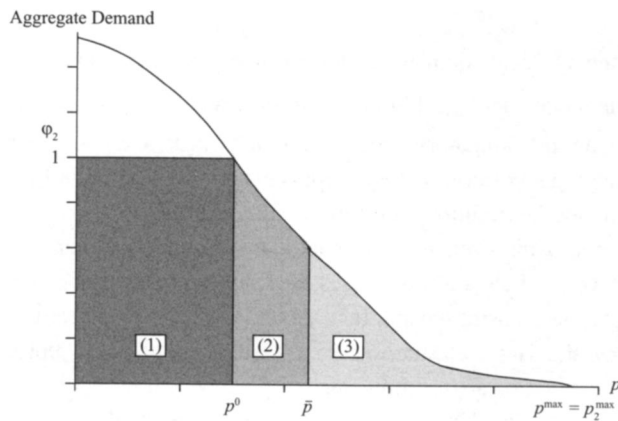


FIGURE 4.3. — α -Uniform Auction (Aggregate Demand)

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p_i^{\max} exceeds \bar{p} . Finally, note that unlike the α -discriminatory format, a slight change in a participant's bid function under the α -uniform format, may not affect seriously the highest price paid by that bidder. Indeed, an individual deviation should only moderately influence the aggregate demand $\Phi(p, s)$. Therefore, \bar{p} as defined in (8) should not vary significantly. To better appreciate the similarities and differences between the two auction formats, consider the following example.

Example 4: Consider the same example as before, except that the auctioneer now selects the α -uniform payment mechanism with $\alpha = 2/3$. The bid functions of the two bidders are plotted in FIGURES 4.1 and 4.2, while the aggregate demand is presented in FIGURE 4.3. As just explained, the highest price p is defined in FIGURE 4.3, such that area 2 corresponds to $2/3$ of the aggregate discriminatory surplus (areas 2 and 3). The highest price p may then be reported on the plot of each bidder's bid function in order to determine their exact payment (i.e. areas 1 and 2 in FIGURES 4.1 and 4.2). Finally, a comparison of FIGURES 2.1 and 4.1 (respectively, 2.2 and 4.2) indicates that, although the two auction formats would generate the same total revenue for the Treasury, bidder 1 (respectively, bidder 2) would pay a slightly higher (respectively, lower) price under the α -uniform pricing rule compared to the α -discriminatory pricing rule.

IV.2. 2. The α -Price-Uniform Auction

The α -price-uniform format is the analog to its α -discriminatory counterpart presented in section 4.1.2, except that once again the highest price paid is common to all bidders. This new highest price \bar{p} is now defined as a convex combination between the stop-out-price p^0 and p^{\max} , the highest price announced across all bidders,

$$\bar{p} = p^0 + \alpha(p^{\max} - p^0) \tag{9}$$

As with the other payment mechanisms previously introduced, the revenue of the Treasury under the α -price-uniform format will lay between the revenue generated under the traditional uniform and discriminatory auction formats. Note however, that for a specific value α (and the same bid functions), the revenue generated by the Treasury is superior under the α -price-uniform format compared to the α -price-discriminatory format. Indeed, \bar{p} (as defined in equation 9) is necessarily larger or equal than \bar{p}_i (as defined in equation 7) for any bidder i .

The α -price-uniform format is a perfect illustration of the fact that with "uniform-type" auctions, the highest price paid by a bidder does not depend directly on its own bid function. Indeed, unless a bidder submits the highest price p^{\max} , he has virtually no control (except indirectly through p^0) over the value of the highest price \bar{p} he will pay. Slight deviations in bidding strategies are then essentially costless. This obviously contrasts sharply with the α -price-discriminatory auction in which \bar{p}_i , the highest price paid by bidder i , depends directly on its own bid function through the highest price he announced \bar{p}_i^{\max} (see equation 7).

Example 5: Consider the same example as before, except that the auctioneer now selects the α -price-uniform payment mechanism with $\alpha = 2/3$ (see FIGURES 5.1 and 5.2). Since $p_1^{\max} < p_2^{\max}$, the highest price paid by each bidder is set to $\bar{p} = p^0 + 2/3(p_2^{\max} - p^0)$. This highest price may then be reported on the two bid functions in order to determine the exact

payment of each participant. Note that since $p_1^{\max} < \bar{p}$, the payment of bidder 1 is in fact equivalent to what he would have paid under the traditional discriminatory format. Bidder 2 on the other hand, pays exactly the same amount as he would have under the α -pricediscriminatory format (i.e., FIGURES 3.2 and 5.2 are exactly the same).

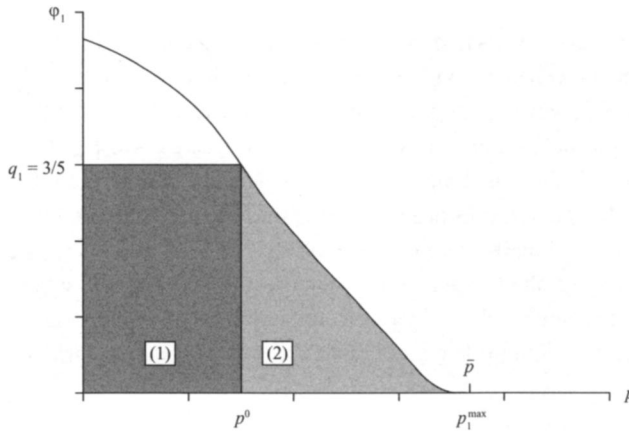


FIGURE 5.1. — α -Price-Uniform Auction (Bidder 1)

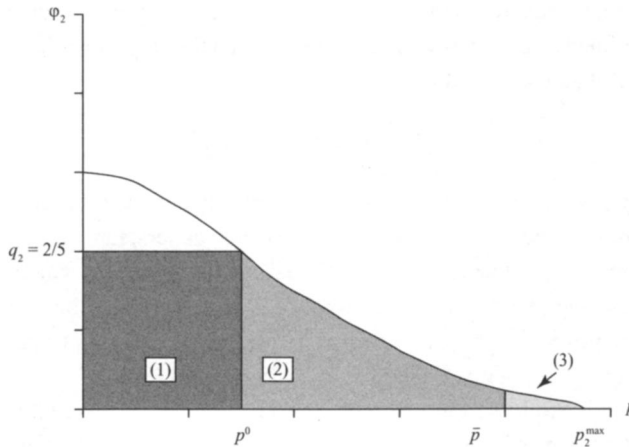


FIGURE 5.2. — α -Price-Uniform Auction (Bidder 2)

IV.2.3. The k^{th} -Average-Price Auction

This pricing rule is inspired by the second-price payment mechanism initially proposed by VICKREY (1961) to conduct single-unit auction. At a single-unit second-price auction, the good is allocated to the highest bidder, and the price paid equals the second highest bid submitted. Although rarely used in practice, the second-price auction possesses interesting theoretical properties. In particular, under the private-values paradigm, it is a dominant strategy for a bidder to bid his true private valuation. This pricing rule has then been generalized into a k^{th} -price auction

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in which the highest bidder pays only the k^{th} highest price submitted. Note, however, that this generalized model does not possess the same theoretical properties as the second-price auction. In particular, truthful bidding is not necessarily an equilibrium strategy under the private-values paradigm (see e.g. WOLFSTETTER 2001).

To explain the k^{th} -average-price auction, let us first define p_i^a the average winning price submitted by a winning bidder i for the share he is allocated:

$$p_i^a = \frac{p^0 \varphi_i(p^0, s_i) + \int_{p^0}^{p_i^{\max}} \varphi_i(p, s_i) dp}{\varphi_i(p^0, s_i)}$$

We can then define $(p_{(1)}^a, p_{(2)}^a, \dots, p_{(n)}^a)$, the vector of average winning prices ranked in decreasing order, for the $n \leq N$ bidders who receive a strictly positive share of the security. The common highest price \bar{p} is then set equal to the k^{th} highest average winning price $p_{(k)}^a$ (and $\bar{p} = p_{(n)}^a$ when $n < k$).

Observe that under the k^{th} -average-price auction truthful revelation is not a dominant strategy. Instead, just like with the other auction formats presented in this paper, participants have an incentive to shade their bids.¹⁷ Moreover, note that once again a slight change in the bid function of any participant but one (the one setting $p_{(k)}^a$) has little bearing on the highest price he will pay, as long as it does not change the identity of the bidder submitting the k^{th} highest average winning bid.¹⁸

4. The Spanish Auction Format

We conclude this section by presenting an additional member of the family of “uniform-type” auctions. This payment mechanism has been used for two decades in Spain to sell Treasury securities. Under this auction format, \bar{p} is defined as the weighted average price of all winning bids:

$$\bar{p} = \frac{p^0 Q + \int_{p^0}^{p^{\max}} \Phi(p, s) dp}{Q}$$

where, as previously defined, Q is the quantity supplied by the Treasury, $\Phi(p, s)$ represents the aggregate demand function as defined in (1), and $p^{\max} = \max_i (p_i^{\max})$ is the highest price announced across all bidders.

The Spanish experience has given rise to a limited number of economic analyses. In particular, using simulations of a simplified model in which two units of Treasury bonds are for sale, ALVAREZ and MAZON (2002) find that the Spanish format may in some cases generate higher revenue than the traditional discriminatory format. ABBINK *et al.* (2006), using a comparable setting, confirm experimentally that the Spanish auction may dominate the traditional discriminatory format. This result has then been slightly generalized by ALVAREZ, CERDA and MAZON (2003). Indeed, adopting a share auction model with linear strategies, the authors find that, for a given set of structural parameters, the Spanish format may increase the Treasury’s revenue over

17. Consider in particular the bidder setting the k^{th} -average-price $p_{(k)}^a$. By bidding truthfully he makes no profit, while shading his bid ensures strictly positive expected profits.

18. To shorten the presentation, and since the mechanics of the k^{th} -average-price and the Spanish auction formats are not directly comparable to the previous pricing rules, we do not provide here graphical illustrations. We refer the reader to our website (http://www.sceco.umontreal.ca/liste_personnel/armantier/papers/Additional_Examples.pdf) for the plots of the bidders payments under the k^{th} -average-price and the Spanish auction formats.

the traditional discriminatory and uniform-price auctions. Our analysis of the Spanish auction format differs in essentially two ways from the papers just mentioned: first, the theoretical model we adopt is significantly richer as it accounts simultaneously for random supply, risk aversion and asymmetry; second, our simulations are conducted with specific structural parameters estimated for two types of securities sold at French Treasury auctions. The counterfactual analysis conducted in section 5 will therefore help us establish whether or not the Spanish auction format would have increased the revenue of the French Treasury in these auctions.

V. Counterfactual Comparison of Payment Mechanisms

We now conduct Monte Carlo simulations to compare the revenue the French Treasury would have generated during the 118 auctions in A&S sample. To do so, we simulate 1,000 times each of these 118 auctions based on i) the structural parameters estimated by A&S, and ii) the exogenous variables corresponding to that auction (*i.e.* the number of bidders in group 1 and in group 2, the security's type, yield and maturity, and the announced bracket for the quantity to be served).¹⁹ Note also that, following A&S, we calculate the Bayesian Nash equilibrium bid function under each of the six payment mechanisms we just presented, using the numerical technique developed by ARMANTIER, FLORENS and RICHARD (2008).²⁰

The results of the counterfactual analysis are presented graphically in FIGURES 6.A to 6.E. These figures illustrate the changes in the revenue of the French Treasury as a function of α and k under the different α -type and k -average-price payment mechanisms. In each figure, we plotted two curves. The first curve (identified by square markers) represents the total expected revenue a given payment mechanism would have generated for the French Treasury during the auctions in our sample. The second curve (identified by round markers) represents the expected standard deviation in the revenue of the French Treasury from one auction to the next.²¹ Finally, in each figure, we differentiated markers with either a full or an empty background. In contrast with a marker with a full background, a marker with an empty background identifies an estimate not significantly different (at the 5% significance level) from the corresponding estimate obtained under the discriminatory format. For instance, on the plot of the total revenue in FIGURE 6.A, an empty square indicates that the revenue generated by the corresponding α -discriminatory auction is not significantly different from the revenue generated by the traditional discriminatory auction format.²²

19. The Monte Carlo simulations rely on the common random number technique. In other words, the comparison of the payment mechanisms are conducted with the same exogenous variables and the same draws for the simulated true values and private signals. As a result, the Monte Carlo simulations may be directly compared across payment mechanisms.

20. More precisely, under each payment mechanism, we must calculate a different equilibrium strategy in each of the 118 auctions in our sample. Indeed, the equilibrium bid function changes from one auction to the next since the exogenous variables, and therefore the characteristics of the auction, are different.

21. The standard deviations in FIGURES 6.A to 6.E are calculated across the 118 auctions in our sample. The standard deviations therefore represent a measure of the variation of the French Treasury revenue from one auction to the next. In other words, these standard deviations should not be confused for a measure of the accuracy of an estimate, as typically presented along with the results of a regression.

22. To shorten the presentation, we have decided not to plot any equilibrium bid function. Indeed, recall that there is a different Bayesian Nash equilibrium strategy for each payment mechanism, each type of players, and each of the 118 auctions in our sample. In addition, according with the plots in A&S, the shapes of the different equilibrium bid functions are mostly flat and essentially uninformative.

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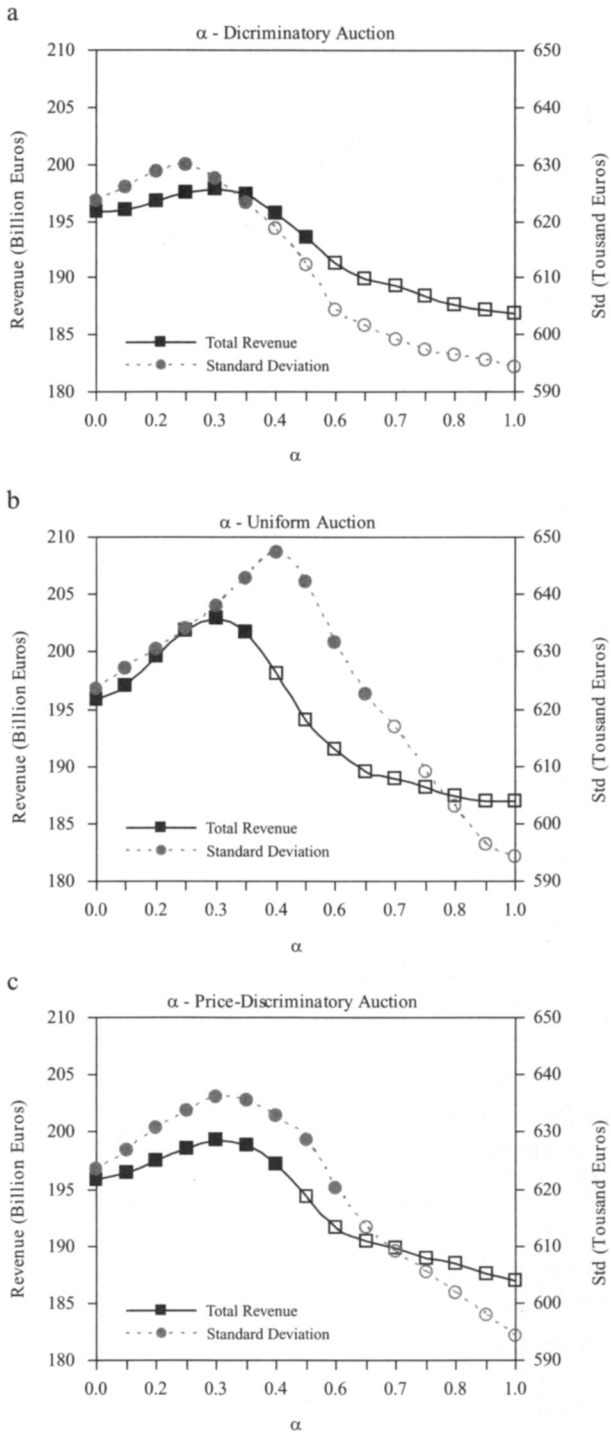


FIGURE 6

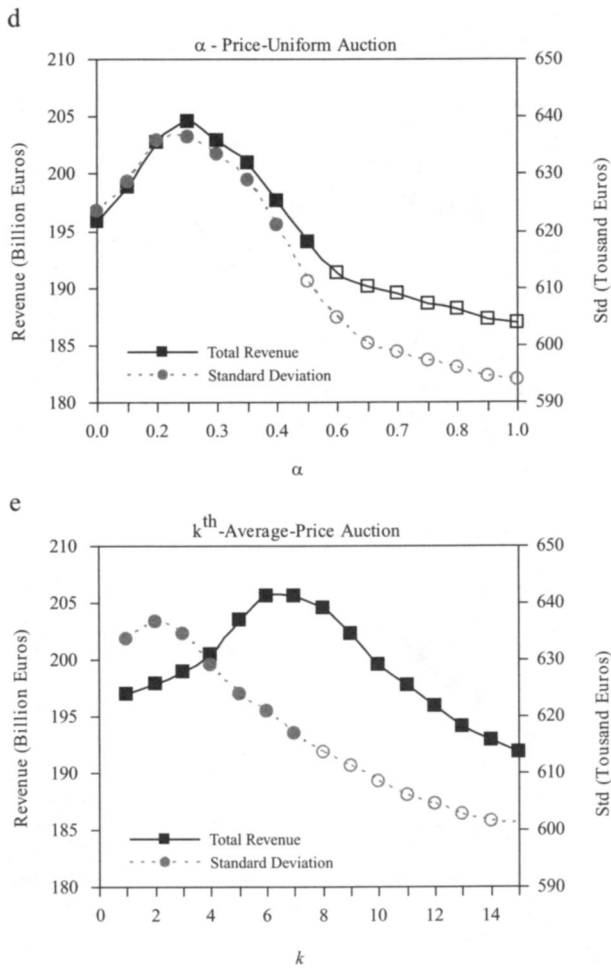


FIGURE 6 – Suite

V.1 Ranking between the Traditional Uniform-Price, Discriminatory and Spanish Formats

We start by comparing the three auction formats that have been used in practice to sell Treasury securities (*i.e.* the traditional uniform-price, discriminatory and Spanish formats). Recall that under any of the α -type payment mechanisms (*i.e.* α -discriminatory, α -price-discriminatory, α -uniform and α -price-uniform), $\alpha = 0$ corresponds to the traditional uniform-price format, while $\alpha = 1$ corresponds to the traditional discriminatory format. The first and last points on FIGURES 6.A to 6.D therefore correspond to the results found in A&S in their analysis of the traditional discriminatory and uniform-price formats. Namely, we estimated that for the auctions in our sample, the discriminatory format (respectively, the uniform-price format) would have generated a total expected revenue of 186,864.9 billion Euros (respectively,

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195,810.3 billion Euros).²³ In other words, the French government would have significantly increased its revenue by nearly 9 billion Euros (or 4.8%) had it used the uniform-price format instead of the discriminatory format.

To understand why the uniform-price auction dominates, it is important to remember that a fundamental characteristic of this format is that bidders do not pay their bid for each unit they receive. Instead, a winning bidder pays a single price, the stop-out-price, for every units he is allocated. As a result, bidders are inclined to announce higher prices than at discriminatory auctions for the first units demanded. Indeed, strategic bidders realize that the stop-out-price they will have to pay will be lower than the price they announced for these initial shares. In particular, A&S find that the group of small bidders is willing to take relatively more risk under the uniform-price auction. Indeed, small banks, which are more risk averse and less informed about the true value of the security, submit larger bids for any relevant quantity under the uniform-price format, as they know that i) they will not have to pay the price they announce, and ii) their bids selection has little bearing the stop-out-price. As a result of this more aggressive bidding behavior, the average price paid by a bank (regardless of type) for a unit of the security increases, which implies that the revenue raised by the French Treasury would have been higher under the uniform-price format.

As noted by A&S however, the uniform-price auction has a drawback. Indeed, as illustrated in (e.g.) FIGURE 6.A, the standard deviation of the per auction revenue is significantly higher under the uniform-price format (623.4 thousand Euros) than under the discriminatory format (594.1 thousand Euros). In other words, the revenue raised by the Treasury is significantly more variable from one auction to the next under the uniform-price format. Therefore, although the Treasury revenue are higher under the uniform-price format, the precise amount of money an auction will generate becomes less predictable. Under these circumstances, it may be more difficult for the French government to use uniform-price auctions as an efficient short-term tool to manage its public debt.

Finally, A&S find that these results are context specific, and they may not extend directly beyond the French Treasury experience. Indeed, Monte-Carlo simulations suggest that alternative values of the structural parameters may yield different conclusions. In particular, A&S find that the discriminatory format would generate higher revenue for the Treasury than the uniform-price auction, if bidders had been found to be symmetric, risk neutral and small banks had received private signals drawn from the same distribution as their larger counterparts. In other words, accounting for informational and risk aversion asymmetries is crucial to determine accurately the payment mechanism generating the highest revenue.

We now turn to the Spanish auction. We find that this format would have generated a total expected revenue of 191,534.3 billion Euros. This amount is significantly larger (respectively, smaller) at a 5% significance level (respectively, 10% significance level) than the corresponding estimate obtained under the discriminatory format (respectively, the uniform-price format).

23. The revenue estimate obtained under the discriminatory format is very close to the 186,749.1 billion Euros actually raised by the French Treasury over the 118 auctions in our sample (see TABLE I). We also show in A&S that, beyond average revenue, our simulated model replicates fairly well various characteristics of the auctions in our sample. For instance, the stop-out-prices, the average price paid by unit, the highest prices at which a positive quantity is demanded, and the repartition of the security between large and small banks are all very similar in the simulations and in the data.

In other words, in terms of the revenue raised by the French Treasury, the uniform-price format dominates the Spanish format, which in turns dominates the discriminatory format. This result is consistent with the theoretic analysis of ALVAREZ *et al.* (2003) and the experimental analysis of ABBINK *et al.* (2006). Note also that the standard deviation of the per auction revenue is significantly lower under the Spanish format (543.4 thousand Euros) than under the two other formats. In other words, the French government may find in the Spanish format a good compromise between raising the highest possible revenue and maintaining a stable stream of revenues from one auction to the next.

V.2 *Alternative Formats*

V.2.1. α -Type Payment Mechanisms

In this section, we consider the α -type auction formats which, to the best of our knowledge, have never been implemented to sell Treasury auctions. We plotted in FIGURES 6.A to 6.D the revenue the French Treasury would have generated under the different α -type payment mechanisms for different values of α . Three comments are in order at this point. First, we assume that prior to any auction, the value of α is fixed by the Treasury and common knowledge. Second, the value of α is assumed to be the same for each of the 118 auctions in our sample. A possible alternative would be for the Treasury to select the value of α prior to each auction as a function of the exogenous variables, such as the yield, the quantity to be served and possibly, the number of bidders. We opted for a fixed α policy as a simplification, as we suspect it would be easier to implement in practice. Third, we do not attempt to determine the exact value of α that maximizes the revenue of the French Treasury. Instead, we consider a grid consisting in common fractions such as deciles and quartiles. This choice was motivated by the fact that the auctions would be easier to implement in practice with a common fraction.

FIGURES 6.A to 6.D indicate that, under the α -type payment mechanisms, the Treasury's total revenue is systematically skewed to the left, and reaches its maximum for α between 0.25 and 0.35. The non-monotonic shape of the Treasury's revenue (*i.e.* first increasing and then decreasing) may be explained by the following trade off. On one hand, a small α induces more aggressive behavior. Indeed, recall that the α -type payment mechanisms get closer to the traditional uniform-price auction when α gets smaller. As a result, bidders, and in particular small bidders, become more aggressive knowing they are less likely to pay the bids they actually submit. On the other hand, the revenue extracted by the Treasury per bidder increases with α . Indeed, recall that α -type payment mechanisms get closer to the traditional discriminatory auction format when α gets close to 1, in which case a bidder must pay, in addition to his payment under the uniform-price auction, almost all of its discriminatory surplus. Our simulations suggest that in the specific case of the French Treasury auctions, the best compromise may be found for low values of α between 0.25 and 0.35. This result may be explained in part by the influential role played by small bidders in the French Treasury auctions under consideration. Indeed, as indicated in TABLE I small bidders outweighed large bidders by a ratio of 3 to 1 on average. In such a context, it is understandable that, to increase its revenue, the French Treasury needs to incite small bidders to be more aggressive and submit larger bids.

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Note also that although it does not dominate for every possible value of α , the α -price-discriminatory and α -price-uniform formats (FIGURES 6.C and 6.D) typically yield higher revenue than respectively the α -discriminatory and α -uniform formats (FIGURES 6.A and 6.B). Likewise, within the class of α -type formats, the uniform-price mechanisms (i.e. α -uniform and α -price-uniform formats in FIGURES 6.B and 6.D) dominate the discriminatory price mechanisms (i.e. α -discriminatory and α -price-discriminatory formats in FIGURES 6.A and 6.C) for most values of α . Overall however, it appears that in our context, the best payment mechanism within the class of α -type formats, is the α -price-uniform payment mechanism, with $\alpha = 0.25$. In this case, the total expected revenue reaches close to 204.5 billions Euros over the 118 auctions in our sample, corresponding to a significant increase of 9.4% compared to the discriminatory format currently used by the French Treasury.

To understand why the α -price-uniform format dominates, it is important to remember that under this payment mechanism a bidder has the least control over the highest price he would have to pay. As a result, bidders, and in particular small bidders, can afford to be more aggressive, which benefits the French Treasury. Indeed, recall that under the α -discriminatory and α -uniform formats, the highest price paid by a bidder depends directly on the entire shape of the bid function he submits. In contrast, under the class of α -price auctions (i.e. α -price-discriminatory and α -price-uniform formats) the highest price paid by a bidder depends almost exclusively on the highest price he submits for the first unit of the security. As a result, bidders, and in particular small bidders, have an incentive to behave more aggressively by submitting steeper bid functions. Moreover, we have seen that, by definition, the highest price paid by a bidder under discriminatory-type auctions depends directly on the bids he submits, while under uniform-type auctions a bidder has very little control over the highest price he will pay.

V.2.2. k^{th} -Average-Price Payment Mechanism

We illustrate in FIGURE 6.E the revenues the French Treasury would have generated under the k^{th} -average-price payment mechanism, for different values of k . Once again, we assume that the French Treasury fixes and announces a value for k prior to the 118 auctions in our sample. This implies in particular that although the number of bidders may vary from one auction to the next, the value of k remains unchanged. As we have seen in Section 3.2, the number of bidders in the French Treasury auctions we analyze is somewhat stable around 19. Therefore, we have calculated the revenues of the French Treasury for k varying from 1 to 15. Since one cannot predict exactly the number of winning bidders, the values of k we consider may turn out to be larger than the number of winning bidders in some of the auctions in our sample. Recall that in such cases, the highest price \bar{p} is set to the lowest average submitted winning bid across bidders.

Observe first that the shape of the French Treasury's revenue as a function of k is somewhat similar to a bell-curve. This result may be explained by the following trade off. On one hand, the French Treasury would like to select k as small as possible. Indeed, all things being equal otherwise, the French Treasury benefits the most when k equals one, since in this case \bar{p} , the highest price paid, is determined by the bidder submitting the highest average bid. On the other hand, a low value of k does not incite the group of large banks, which typically submit the highest bids, to be aggressive. Indeed, \bar{p} is in this case likely to be determined by one of

these large banks. In contrast, when k is high, the large banks can afford to submit high bids as these are unlikely to influence \bar{p} . Indeed, \bar{p} is in this case likely to be the average winning bid of one of the small banks. We can see in FIGURE 6.E that the best compromise between setting \bar{p} high and inciting large bidders to be aggressive is found for $k = 6$. In that setting, the French Treasury revenue would have been 205.6 billions Euros over the auctions in our sample, 10% higher than the revenue generated by the discriminatory format. This 6th-average-price auction format also dominates, although not always significantly, the family of α -type payment mechanisms we just discussed.²⁴ Intuitively, this result may be explained by the fact that, as mentioned in Section 4.2.3, bidders, and in particular the bidders submitting the highest bids, can afford to be aggressive as the bid function they submit is likely to have little or no weight on the characterization of \bar{p} , the highest price they pay. To conclude, it is worth noting that the standard deviation of the per auction revenue is significantly higher under the 6th-average-price mechanism than under the discriminatory auction format used by the French Treasury. In other words, the discriminatory format may still be preferred if the objective of the French government is to generate a stable stream of revenues to finance its debt.

VI. Conclusions

The aim of this paper was to propose and investigate new Treasury auction payment mechanisms, as a possible alternative to the traditional discriminatory and uniform-price formats. Using the structural model estimated by ARMANTIER and SBAÏ (2006), we conduct a counterfactual analysis to rank these different formats in terms of the revenue they would have raised for the French Treasury over the 118 auctions in our sample. When comparing the payment mechanisms that have been actually implemented to sell Treasury securities, we find that the uniform-price format dominates the Spanish auction, which in turns dominates the discriminatory format currently used by the French Treasury. This result is consistent with the analysis conducted by ABBINK *et al.* (2006). Among the alternative payment mechanisms we proposed, we find that the format generating the highest revenue for the French Treasury is the k^{th} -average-price format, with $k = 6$. During the two and a half years covered by our sample, the 6th-average-price auction format would have raised the French Treasury revenue by about 10 billions euros (or 4.7%) over the uniform-price format, and by about 19 billions euros (or 10.0%) over the discriminatory format.

The 6th-average-price auction format however, may suffer from two potential drawbacks. First, one may wonder whether such a payment mechanism would be well understood and/or accepted by the participants at French Treasury auctions. We should note however, that this format is not necessarily more complex to implement than the Spanish format which has been used in Spain since January 1987 without notable problems or complaints. The second potential drawback is that the revenue generated from one auction to the next by the 6th-average-price auction format is more variable than under other payment mechanisms. In particular, although it generates a per auction revenue 6.8% lower, the Spanish format is 12.5% less variable from one

24. In particular, the revenues generated by the 6th-average-price auction and the best α -type format (the α -price-uniform payment mechanism with $\alpha = 0.25$), cannot be distinguished statistically at the usual significance levels.

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auction to the next than the 6th-average-price auction. In other words, the Spanish auction format may be preferred if the objective of the government is to generate a stable stream of revenues in order to better manage its debt in the short term.

Moreover, we must acknowledge some of the limitations of the model we adopted. In particular, we have assumed that the participation at a French Treasury auction was exogenous. It is possible however, that the number of bidders may change depending on the value of the exogenous variables, and/or on the payment mechanism adopted. The analysis of such a model, is significantly more challenging, as it would require the estimation of a model with endogenous participation. To the best of our knowledge, such a model does not exist in the literature on Treasury auctions.

Finally, note that the conclusions of this paper may not immediately extend to Treasury markets in other countries. Nevertheless, our analysis may be considered of general interest. Indeed, just like in France, many Treasury auctions around the world seem to involve asymmetric bidders, and are conducted under the discriminatory format. Therefore, the methodology developed by ARMANTIER and SBAI (2006) could first be applied to estimate the structural parameters underlying the treasury auctions in a specific country. Then, a counterfactual analysis similar to the one conducted here could be implemented to determine which payment mechanism appears to be the most advantageous to the Treasury in that country.

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